Capital Allocation, Portfolio Enhancement and Performance Measurement : A Unified Approach *)

Winfried G. Hallerbach **)

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**) Dept. of Finance, Erasmus University Rotterdam, POB 1738, NL-3000 DR Rotterdam, The Netherlands, phone +31.10.408-1290, facsimile +31.10.408-9165, hallerbach@few.eur.nl , home page: http://www.few.eur.nl/few/people/hallerbach/, see also http://www.finance-on-eur.nl.

Abstract

Risk analysis, economic capital allocation and performance evaluation are crucial steps in the process of enterprise-wide risk management. Capital-at-Risk (CaR) plays a central role since it determines the amount of economic capital that is required to support firm-wide consolidated risks and it is the key ingredient of risk-adjusted return (RAROC) measures. The existing literature, however, offers various definitions of RAROC. In addition most approaches assume a joint-elliptical world. Especially in the context of credit risk, where loss distributions are skewed, this is not realistic. Moreover this leads to biases in estimating the risk contributions of portfolio components and in determining the subsequent allocation of economic capital.

In this paper we study capital allocation and risk-adjusted performance measurement (RAPM) in a coherent and non-parametric framework. Our results can readily be used in a simulation context and serve as a benchmark to evaluate the corresponding *CreditMetrics*, *CreditRisk+* and *KMV* approaches. We first discuss the allocation of economic capital over business units or portfolio components according to their risk contributions. We then show that the relevant RAROC measure, based on relative risk-return contributions, actually *emerges* from the solution to a suitable CaR-constrained portfolio optimization problem. This implied RAROC is important as a decision measure for shaping portfolio composition *ex ante facto*; as a performance measure it serves to evaluate and to attribute portfolio performance *ex post facto*. However, different decision problems imply different RAROC measures. The relevant definition of a RAROC measure depends on the specific decision context at hand and, consequently, no generally valid recipes can exist. Hence we propose a unified approach to portfolio optimization, economic capital allocation and RAPM.

Key words: Capital-at-Risk, risk-adjusted performance evaluation, RAROC, portfolio optimization, non-parametric methods

JEL classification: C13, C14, C15, D81, G11, G20

1. Introduction

Market risks, credit risk and operational risks are the main risk categories faced by financial institutions. To an increasing degree the allowed exposures from these risks are subject to regulation (notably by the "Basle II" proposals). Under the denominator of "Enterprise-Wide Risk Management" (henceforth EWRM) these risks are analyzed in a coherent way. This is a challenging task.[1]

Financial institutions hold reserves and provisions in order to cover expected losses incurred in the normal course of business. In order to provide a cushion against unexpected losses they must hold some amount of capital. The minimum amount of capital required by BIS regulations is termed regulatory capital. Financial firms also specify internal capital requirements in order to ensure solvency. The minimum amount of internal capital is termed *economic capital*. Defined as a one-sided confidence interval on potential portfolio losses over a specific horizon, Value-at-Risk (VaR) serves the role of setting the capital requirement for market risks. Because of the frequent portfolio revisions the VaR horizon is chosen fairly short, ranging from one to twenty trading days. In the context of credit risk VaR is often denoted as Credit-VaR; in the general context of enterprise-wide risk the VaR measure is termed Capital-at-Risk (CaR). These metrics serve to set the amount of economic capital. Compared to VaR the focus is more on solvency than on liquidity so that the horizon is longer, typically one year.[2] Also, since the continuity (non-default) of the firm is at stake the confidence level is set fairly high, typically 99% or even 99.5%.[3] For simplicity we henceforth gather the concepts of VaR, Credit-VaR and CaR under the generic term of CaR.

Probabilistic analyses of potential portfolio losses date back more than a century. An analysis of CaR *avant la lettre* is provided by Edgeworth [1888] who invoked the central limit theorem and used quantiles of the normal distribution to analyze potential bank losses and to evaluate bank solvency. Motivated by the BIS' proposals and the EU's Capital Adequacy Directives, the release of *RiskMetrics*™ by J.P.Morgan [1994] in October 1994 spurred the development of the VaR concept. Nowadays a wide variety of analytical and simulation-based estimation methods is available for market risk and credit risk analysis.[4,5] For a critical overview of recent developments, see for example the special issue of the *Journal of Banking & Finance* [2002].

Extending the VaR concept from a trading environment to a credit risk – and more general, an EWRM – context raises some interesting problems. EWRM entails several steps, viz. risk analysis, portfolio optimization, economic capital allocation, and risk-adjusted performance (RAP) evaluation. In the first step the market and credit risks must be analyzed in a consistent way, recognizing the interdependency of these risks. The parametric assumption of symmetrical (viz. elliptical) distributions may be

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[1] See for example Bookstaber [1997] and Cumming & Hirtle [2001].

[2] Horizon issues are discussed in Kupiec [1999] and Shen [2001].

[3] Such high confidence level renders model validation by backtesting virtually impossible, especially when combined with a long horizon. See also footnote 2.

[4] The choice for a specific estimation method depends on both the degree of non-linearity of the instruments comprised in the portfolio and the willingness to make restrictive assumptions on the underlying statistical distributions. See for example Duffie & Pan [1997] and Jorion [2001] for an overview.

[5] Kupiec [2001] discusses estimating Credit-VaR *vis à vis* different applications.

analytically convenient in CaR analyses[6], but it is not appropriate. The decomposition of portfolio CaR with respect to individual activities is then troublesome and may distort the allocation of economic capital over portfolio components, portfolio optimization, and the RAP analysis. In CreditSuisse's *CreditRisk+* [1997], for example, the allocation is performed proportionally to standard deviation and this approach breaks down when the implied ellipticity assumption is violated.[7] In KMV's [1997] portfolio optimization procedure a conventional Sharpe [1966,1994] ratio is used, which is also biased when returns are not elliptical. Also the performance measurement on the basis of conventional RAROC measures is troublesome. Firstly because these measures may be based on an unsuitably parametrically estimated CaR. Secondly because throughout the literature RAROC measures are *defined* and not derived.[8] Hence there exists substantial ambiguity in *ex cathedra* proposed RAP measures. We here argue that the relevant risk-adjusted performance measure is *implied* by the underlying optimization problem. Hence the relevant definition of a RAROC measure depends on the specific decision context at hand. Consequently no generally valid recipes can exist.

In this paper we study portfolio optimization, capital allocation and risk-adjusted performance measurement (RAPM) in a coherent and non-parametric framework. Our results can readily be used in a simulation context and serve as a benchmark to evaluate the corresponding *CreditMetrics*, *CreditRisk+* and *KMV* approaches.[9]

The outline of the paper is as follows. In section 2 we briefly review the concepts of economic capital, CaR, and RAROC, and we discuss the allocation of economic capital over business units or portfolio components according to their risk contributions. In section 3 we analyze portfolio optimality and RAPM in some simplified decision contexts. We show that the implied portfolio optimality conditions guide the choice of the appropriate RAROC metric. More specifically we show that the relevant RAROC measure, based on relative risk-return contributions, actually *emerges* from the solution to the suitable CaR-constrained portfolio optimization problem. This implied RAROC is important as a decision measure for shaping portfolio composition *ex ante facto*; as a performance measure it serves to evaluate and to attribute portfolio performance *ex post facto*. Hence we propose a unified approach to portfolio optimization, economic capital allocation and RAPM. In practice, there are restrictions on portfolio revisions and flexibility in portfolio composition is limited. Attention thus shifts from fully-fledged portfolio optimization to portfolio enhancement. In this context our results can be used *(i)* to estimate the risk-return trade-off that is implied by a given sub-optimal portfolio, *(ii)* to gauge the degree of its sub-optimality, and *(iii)* to improve the portfolio in accordance with the estimated risk-return trade-off. Section 4 concludes the paper and presents lines for further research. The Appendix contains technical details.

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[6] See for example Saita [1999], Stoughton & Zechner [1999,2000] and Dowd [1998,1999,2000] who assume normality throughout. In response, Tasche & Tibiletti [2001] relax this assumption by investigating suitable approximations. In Hallerbach [2003] we also present a non-parametric approach.
[7] See Hallerbach [2003]. For correct decomposition procedures in *CreditRisk+* see Haaf & Tasche [2002] and Kurth & Tasche [2003].
[8] See Bessis [2002] and Matten [2000], e.g.
[9] For a comparison of these Credit-VaR models we refer to Crouhy, Galai & Mark [2000].

2. Preliminaries

In this section we introduce notation and present some useful results. We first discuss the theoretical concepts of overall CaR, marginal CaR and component CaR.[10] Since we want to discuss CaR and RAROC in the most general context, the only (and very weak) assumption we make is that all relevant return distributions have finite first moments. We then summarize the concept of RAROC and outline its role in RAPM.

defining overall CaR

Consider a portfolio p with current value V_p, consisting of N components. In the broad context of EWRM the portfolio is the overall firm and the components represent the partitioning of the firm's business activities according to separate activities, organized activities or business units, e.g. In the context of credit risk the components are the individual credits or loans comprised in the portfolio. Given the current values $\{V_i\}_{i\in p}$ of the dollar positions in each of the component activities, the change in portfolio value over a holding period Δt equals:

$$(2.1) \quad \Delta\tilde{V}_p \equiv V_p\,\tilde{r}_p = \sum_{i\in p} V_i\,\tilde{r}_i \qquad \text{with :} \qquad \sum\nolimits_{i\in p} V_i = V_p$$

where $\tilde{r}_p$ and $\tilde{r}_i$ denote the Δt return on the portfolio and activity i, respectively. A tilde marks a stochastic variable.[11] All returns denote *total returns* and reflect both changes in market value ("price return") and cash flows ("cash return") during the period.[12] The portfolio composition is assumed constant over the period Δt.

Given the portfolio p, its expected dollar return is:

$$(2.2) \quad \Delta\overline{V}_p \equiv V_p\overline{r}_p = \sum_{i\in p} V_i\overline{r}_i$$

where $\overline{r}_i$ is the expected percentage return on activity i. Given a confidence level c and an evaluation horizon of Δt, we define the quantile dollar return ΔV_p^c and the quantile percentage return r_p^c (given V_p) that satisfy the specified confidence level:

$$(2.3) \quad \begin{aligned} &\Delta V_p^c \;:\; \Pr\left\{\Delta\tilde{V}_p \le \Delta V_p^c = r_p^c\,V_p\right\} = 1-c \\ &r_p^c \;\;\;\;\;:\; \Pr\left\{\tilde{r}_p \le r_p^c\right\} = 1-c \end{aligned}$$

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[10] The following results (and notably the correspondence between CaR contributions and conditional expectations) have been first derived by Hallerbach [2003] and Tasche [1999].

[11] We define the variates $\{\tilde{V}_i\}$ and $\tilde{V}_p$ on the probability space $(\Omega, \mathscr{F}, \Pr(\cdot))$, where the σ-field $\mathscr{F}$ contains subsets of the sample space Ω.

[12] When a market value is not available, for example for non-traded credits, the mark-to-market valuation is replaced with mark-to-model valuation. This introduces model risk; cf. Kato & Yoshiba [2000].

Overall portfolio CaR is now given by $CaR_p \equiv -\Delta V_p^c = -r_p^c V_p$ since CaR is defined in terms of losses.[13] When V_p is initially given, focus can be on r_p^c instead of on ΔV_p^c.

defining marginal expected return and marginal CaR

When studying portfolio optimality in section 3 we need information about the marginal expected returns and the marginal CaRs of the individual securities comprised in the portfolio.

From (2.2) it follows that marginal expected return of security i is:

$$(2.4) \quad \frac{\partial(\overline{r}_p V_p)}{\partial V_i} = \overline{r}_i \qquad \forall i \in p$$

The marginal CaR $MCaR_{ip}$ is the change in portfolio CaR resulting from a marginal change in the dollar position in component activity i :

$$(2.5) \quad MCaR_{ip} \equiv \frac{\partial\, CaR_p}{\partial V_i} \qquad\qquad i \in p$$

Note that eq.(2.5) also applies to an activity N that has not yet been included in the firm's portfolio. The initial portfolio p then comprises $N{-}1$ component activities and we consider this ($N{-}1$)-element portfolio as an N-element portfolio where $V_N = 0$ initially.

To evaluate marginal CaR we start from eq.(2.1) which identifies the portfolio dollar return as a convex combination of the dollar returns on the individual components. Because of the portfolio partitioning and by the very definition of conditional expectations we have:

$$(2.6) \quad \Delta\tilde{V}_p = E\left\{\tilde{r}_p V_p \middle| \Delta\tilde{V}_p\right\} = \sum_{i\in p} V_i \tilde{E}\left\{\tilde{r}_i \middle| \Delta\tilde{V}_p\right\}$$

Note that the conditional expectation $\tilde{E}\left\{\tilde{r}_i \middle| \Delta\tilde{V}_p\right\}$ is a random variable.[14] Also note that the portfolio dollar return is a linearly homogeneous function of the positions $\{V_i\}_{i\in p}$. Since this function is continuous and analytic we can apply Euler's theorem:

$$(2.7) \quad \Delta\tilde{V}_p = \sum_{i\in p} V_i \frac{\partial\,\Delta\tilde{V}_p}{\partial V_i} = \sum_{i\in p} V_i\, \tilde{r}_i$$

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[13] We obtain changes in dollar values by combining returns with mark-to-market values. Whether focusing on returns or on dollar positions, transactions with zero initial value (such as credit swaps) have to be decomposed into non-zero long and short positions (mapping). The firm's overall portfolio will have strict positive initial value.

[14] Hence $\tilde{E}\{\tilde{r}_i \,|\, \Delta\tilde{V}_p\}$ is to be interpreted as the expectation of $\tilde{r}_i$ conditional to the σ-field $\mathscr{F}$ relative to which $\Delta\tilde{V}_p$ is defined. For details, see Spanos [1986].

Substituting eq.(2.7) in (2.6) and conditioning on $\Delta\tilde{V}_p = \Delta V_p^c$ yields:

$$\text{(2.8)} \quad CaR_p = -\Delta V_p^c = -\sum_{i \in p} V_i \, E\left\{ \frac{\partial \Delta \tilde{V}_p}{\partial V_i} \,\middle|\, \Delta\tilde{V}_p = \Delta V_p^c \right\} = -\sum_{i \in p} V_i \, E\left\{ \tilde{r}_i \,\middle|\, \Delta V_p^c \right\}$$

where we have added a minus sign since CaR is defined in terms of losses. Since the portfolio return now takes the particular value ΔV_p^c the conditional expectations become deterministic. Under mild regularity conditions we can interchange the expectations integral and the derivative in the second equality of eq.(2.8). Hence, the conditional expectation represents $\partial \Delta V_p^c / \partial V_i$: minus the marginal CaR of i, $MCaR_{ip}$. From the third equality it then follows that:

$$\text{(2.9)} \quad MCaR_{ip} = -E\left\{ \tilde{r}_i \,\middle|\, \Delta V_p^c \right\}$$

Following complementary reasoning, Tasche [1999] independently derived the same result in a VaR context. For formal proofs we refer to Tasche [1999] and Gouriéroux et al. [2000]. The intuition behind eq.(2.9) is clear. When there is a positive (negative) interdependence between $\Delta\tilde{V}_i$ and $\Delta\tilde{V}_p$ then large negative portfolio returns will on average be associated with large negative (positive) component returns. Increasing (decreasing) the size of the activity position V_i will then lower the portfolio value even more, thus increasing the portfolio's CaR.

defining and relating component CaR

Since:

$$\text{(2.10)} \quad CaR_p = -\sum_{i \in p} V_i \, E\left\{ \tilde{r}_i \,\middle|\, \Delta V_p^c \right\} = \sum_{i \in p} V_i \, MCaR_{ip}$$

each term *i* measures the total contribution of asset *i* to the overall portfolio VaR. Hence $V_i \cdot MCaR_{ip} = CCaR_{ip}$ is the component CaR of activity *i.* These component CaRs can uniquely be attributed to each of the individual components of that portfolio and aggregate linearly into the total diversified portfolio CaR:

$$\text{(2.11)} \quad CaR_p \equiv \sum_{i \in p} CCaR_{ip} = \sum_{i \in p} V_i \, MCaR_{ip}$$

Note that because of return interdependencies and diversification effects the components' stand-alone CaRs do *not* add up to the diversified portfolio CaR.[15, 16]

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[15] The break-down of VaR according to portfolio components or market risk factors as suggested by Fong & Vasicek [1997], for example, suffers from this shortcoming and is hence not useful. For a break-down of VaR according to an underlying factor model, see Hallerbach & Menkveld [2003]. For an evaluation of different decomposition methods we refer to Koyluoglu & Stoker [2002].

In Hallerbach [2003] we show how marginal and component CaRs can be estimated in a non-parametric context.

Eq.(2.11) is a powerful result. It does not depend on any distributional assumptions but prevails since the portfolio operator is linear. Without loss of generalization the component activities may be mapped in a non-linear fashion onto standardized positions or underlying state variables (such as default processes or recovery rates, as in J.P.Morgan's [1997] *CreditMetrics*™).

economic capital, RAPM and RAROC

Since economic capital is necessary to cover potential losses from the firm's activities positions $\{V_i\}_{i \in p}$, the RAP is measured by relating generated income to economic capital. The resulting RAP metric, termed RAROC, was first proposed by *Bank of America*.[17] It takes the form:

(2.12) $$RAROC = \frac{adjusted\ income}{CaR}$$

The denominator is "risk-adjusted" or economic capital. On the aggregate level this is the firm's equity and the CaR's confidence level is in line with the firm's default probability. In the numerator income is revenues minus costs minus expected losses. The adjustment for expected loss is generally considered as a risk correction (although it is a provision for *expected* losses, which by definition does not represent risk). For this reason, (2.12) is sometimes called a RARORAC (risk-adjusted return on risk-adjusted capital) measure. Various specific definitions exist (such as RAROC, RARORAC and RORAC), but most variations are due to the specification of the numerator.

The numerator indeed raises some questions. Should financing (opportunity) costs be taken into account? The numerator then represents the "economic profit". More importantly we ask ourselves why the focus is on cash income? Return can also be generated from capital gains or losses? Defining income on a mark-to-market basis can correct for this. In *ex post* applications, how is the numerator measured? Since risk is involved we would like to specify the numerator as an expected return. In *ex post* applications the fair game assumption could be invoked to estimate the expected value by means of an historical average.

The denominator raises the issue of how to allocate the amount of economic capital that relates to the overall diversified firm portfolio over the different sub-levels within the firm (ranging from business units, via geographical locations to trading desks, individual traders and ultimately to individual business transactions).

These ambiguities call for a fool proof definition of RAROC. However, in the next section we argue that the relevant definition of the RAP measure depends on the decision context, comprising the pursued objective(s) and the imposed constraints.

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[16] The sum of the stand-alone VaRs/CaRs can be larger than the portfolio VaR/CaR but also smaller. The latter phenomenon indicates that VaR is not "sub-additive". Hence, VaR is not a coherent risk measure; see Artzner et al. [1999]. Denault [2001] translates the formulated coherence requirements to capital allocation measures.

[17] See Zaik, Walter & Kelling [1996]. For general exposés we refer to Bessis [2002], Matten [2000] or Smithson & Hayt [2001].

3. Portfolio optimization, RAROC and RAPM

In this section we show how the relevant definition of risk contributions and various RAROC measures are implied by a portfolio optimization model. Each RAROC measure is relevant within the specific underlying optimization context. We assume that the firm strives to maximize the expected return on its activities portfolio subject to a constraint on the required economic capital. Economic capital is measured by the firm's CaR over a given horizon. We classify the possible models using two dimensions. The first dimension is defined by the scale of operations. The capital invested in the firm activities (or the budget available for the firm's business ventures) V_p may either be fixed or free. In the former case the available capital is restricted (parallelling a standard portfolio investment problem); in the latter case the firm may increase (decrease) the scale of its activities by raising more (less) capital. The second dimension is defined by the type of CaR constraint.[18] This constraint on economic capital may be formulated in either absolute or relative terms. The absolute CaR constraint is given by the portfolio dollar CaR level $CaR_p^{\max}$ that should not be exceeded. The relative constraint is defined in percentage terms $-r_p^{c,\max}$ of V_p (see eq.(2.3)). The possible combinations are summarized in Exhibit 1.

Exhibit 1: A simple typology of CaR-constrained optimization problems

firm's capital	CaR restriction	
	absolute ($)	relative (%)
V_p free	**I:** max $\Delta\bar{V}_p = \sum \Delta\bar{V}_i$ s.t. $\Delta V_p^c \geq -CaR_p^{\max}$	**IV:** max $\Delta\bar{V}_p = \sum \Delta\bar{V}_i$ s.t. $\Delta V_p^c \geq V_p \cdot r_p^{c,\max}$
V_p fixed	**II:** max $\Delta\bar{V}_p = \bar{r}_p V_p$ s.t. $\Delta V_p^c \geq -CaR_p^{\max}$ $V_p \geq \sum V_i$	**III:** max $\Delta\bar{V}_p = \bar{r}_p V_p$ s.t. $\Delta V_p^c \geq V_p \cdot r_p^{c,\max}$ $V_p \geq \sum V_i$

In all cases we may wish to restrict short positions, $V_i \geq 0, \forall i \in p$. In that case Kuhn-Tucker conditions will apply and only positive positions are considered.[19] When V_p is

[18] There is some debate whether CaR should be discounted over the horizon or not. When CaR should cover potential losses at the end of the horizon the discounting argument is clear. When also intermediate losses should be covered the case is not clear. In the following we refrain from discounting CaR, but the necessary adjustment is obvious.

[19] Throughout the paper we assume that second order conditions are satisfied. Hence we assume that the feasible CaR region is convex.

fixed, $CaR_p^{\max} \Leftrightarrow -r_p^{c,\max} \cdot V_p$ so the optimization problems II and III are equivalent. When the CaR restriction is formulated in relative terms and V_p is not fixed, as in IV, the problem becomes indeterminate and can only be solved for $\{V_i\}$ given some level of V_p. Actually, IV is not realistic under our simple assumptions since in practice there will be some limit to the firm's activities anyhow. Should we allow for a trade-off between expected return and CaR, or when we would add other restrictions, IV would become a relevant starting point. But for now we are left with two different cases: on the one hand we have problem I, and on the other problems II and III.

Another aspect that proves to be important is whether riskfree activities (riskfree borrowing and/or lending) are available to the firm. In section 3.1 we assume that all portfolio components are risky, so there does not exist a riskfree rate. In section 3.2 we drop this assumption and allow for riskfree investment opportunities.

3.1 Portfolio optimization without riskfree rate

problem I

In situation I the firm strives to maximize the expected dollar return over the chosen time horizon subject to an absolute CaR constraint. The risk and hence the economic capital of the firm's activities is restricted by the maximum admissible CaR level $CaR_p^{\max}$. The optimization problem becomes choosing $\{V_i\}_{i \in p}$ such that:

$$(3.1) \quad \max_{\{V_i\}} \ \Delta \overline{V}_p \qquad s.t. \ \ \Delta V_p^c \geq -CaR_p^{\max}$$

We can safely assume that there exist sufficient profitable business ventures so that the CaR constraint is binding. Hence the maximum allowed amount of economic capital will be employed. Forming the Lagrangian and taking the partial derivatives to V_i leads to the following first order conditions (FOCs henceforth):

$$(3.2) \quad \overline{r}_i - \lambda \cdot MCaR_{ip} = 0 \qquad \forall i \in p^*$$

together with the original CaR constraint. λ is the Lagrange multiplier and an asterisk refers to the optimum. Multiplying with V_i and summing over $i \in p^*$ yields, in combination with (3.2):

$$(3.3) \quad \frac{\Delta \overline{V}_i^*}{CCaR_{ip^*}} = \frac{\Delta \overline{V}_j^*}{CCaR_{jp^*}} = \frac{\Delta \overline{V}_{p^*}}{CaR_{p^*}} \qquad \forall i, j \in p^*$$

provided that $MCaR_{ip^*} \neq 0, \forall i \in p^*$. Eq.(3.3) is the *portfolio optimality condition*. When the portfolio is optimal the ratio of marginal (total) return contribution and marginal (total) CaR contribution is constant over all activities in p^*.

To allow for zero marginal CaRs we rewrite (3.3) as:

$$(3.4)\qquad \Delta\overline{V}_i^* = \frac{CCaR_{ip*}}{CaR_{p*}}\Delta\overline{V}_{p*} \qquad \forall i \in p*$$

Optimal allocation of capital is achieved when (3.3) (or (3.4)) is satisfied. Note that for each activity its expected dollar return should be related to its total *contribution* to the diversified portfolio CaR.

In the last term of (3.3) we recognize the familiar firm-wide RARORAC (or RAROC). The numerator is the expected dollar return on the activity portfolio. By definition this return *(i)* is net of the expected loss and *(ii)* includes price returns (capital gains/losses). In the conventional definition, only the cash return (i.e. income) is considered (contrasting *(ii)*) and subtracting the expected loss is meant to yield the "risk-adjusted return". Since the expected loss is expected by definition, this is not a risk correction at all.

The first term of (3.3) indicates how to appraise the *ex ante* performance of activity *i*: by relating its expected dollar return to its contribution to overall economic capital. Now suppose that given some portfolio *p* we find that:

$$(3.5)\qquad \frac{\Delta\overline{V}_i}{CCaR_{ip}} > \frac{\Delta\overline{V}_p}{CCaR_p} > \frac{\Delta\overline{V}_j}{CCaR_{jp}} \qquad i, j \in p$$

Obviously *p* is not optimal. This implies that *p* can be enhanced by increasing the position in activity *i* and decreasing the position in *j*. *Ex ante* performance analysis is thus relevant for evaluating the optimality of some (initial) portfolio and deriving portfolio revision recipes. Under a fair game assumption activity *i*'s *ex post* performance can be gauged by relating its average realized dollar return to its contribution to overall economic capital.

In practice the prescription is to maximize the firm's RAROC. As the Appendix shows, maximizing RAROC yields the same FOCs eqs.(3.3) and (3.4). Obviously the unconstrained maximization of conventional RAROC assumes the underlying optimization problem (3.1). Conversely, maximizing RAROC can be justified on the basis of (3.1).

But suppose now that the firm's total capital V_p is fixed, or that riskfree ventures exist. Obviously the portfolio optimality conditions will change – and hence the implied risk-adjusted performance measure. This is investigated below.

problems II and III

In situations II and III the total available capital V_p is fixed and the firm strives to maximize the expected dollar return over the chosen time horizon subject to an absolute or relative CaR constraint. The optimization problem now becomes choosing $\{V_i\}_{i\in p}$ such that:

$$(3.6)\qquad \max_{\{V_i\}} \quad \Delta\overline{V}_p = \overline{r}_p V_p$$

$$s.t. \quad \Delta V_p^c \geq -CaR_p^{\max} = V_p \cdot r_p^{c,\max}$$

$$V_p \geq \sum V_i$$

From the Lagrangian the FOCs are:

$$(3.7) \quad \overline{r}_i - \lambda \cdot MCaR_{ip} - \theta = 0 \qquad \forall i \in p^*$$

together with the original constraints. λ and θ are the Lagrange multipliers of the CaR and the capital constraints, respectively. Multiplying with V_i and summing over $i \in p^*$ yields, in combination with (3.7):

$$(3.8) \quad \frac{\Delta\overline{V}_i^* - \theta V_i^*}{CCaR_{ip^*}} = \frac{\Delta\overline{V}_j^* - \theta V_j^*}{CCaR_{jp^*}} = \frac{\Delta\overline{V}_{p^*} - \theta V_{p^*}}{CaR_{p^*}} \qquad \forall i, j \in p^*$$

with $CaR_{p^*} = CaR_p^{\max}$, provided that $MCaR_{ip^*} \neq 0, \forall i \in p^*$. This implied portfolio optimality condition is equivalent to:

$$(3.9) \quad \frac{\overline{r}_i - \theta}{MCaR_{ip^*}} = \frac{\overline{r}_j - \theta}{MCaR_{jp^*}} = \frac{\overline{r}_{p^*} - \theta}{-r_{p^*}^c} \qquad \forall i, j \in p^*$$

When the activity portfolio is optimal the ratio of marginal (total) *adjusted* return contribution and marginal (total) CaR contribution is constant over all activities in p^*. Expected returns are adjusted with a factor θ (the shadow price of relaxing the capital constraint) indicating the percentage opportunity cost of obtaining additional funds. To allow for zero marginal CaRs we rewrite (3.8) as:

$$(3.10) \quad \Delta\overline{V}_i^* - \theta V_i^* = \frac{CCaR_{ip^*}}{CaR_{p^*}}\left[\Delta\overline{V}_{p^*} - \theta V_{p^*}\right] \qquad \forall i \in p^*$$

Optimal allocation of the available restricted capital is now achieved when (3.8) (or (3.9) or (3.10)) is satisfied.

The last term of (3.8) (or (3.9) in percentage terms) is the relevant implied risk-adjusted performance metric. The numerator is the expected adjusted return on the activity portfolio. Again, this return *(i)* is net of the expected loss and *(ii)* includes price returns (capital gains/losses). Moreover it is *(iii)* adjusted for the implied shadow cost θ of obtaining additional funds. The first term of (3.8) indicates how to appraise the performance of activity *i*: by relating its expected or average *adjusted* dollar return to its contribution to overall economic capital. From an *ex ante* perspective, deviations for the FOCs can be used to guide portfolio revisions in order to enhance the sub-optimal portfolio. When the adjustment sub *(iii)* is ignored, optimal allocation is not guaranteed. Likewise, *ex post* performance analysis is distorted. Conventional RAROC analysis on the basis of (3.3) for the portfolio, or the individual activities comprised therein, will fail in this case. As shown in the Appendix, the unconstrained maximization of conventional RAROC is at odds with the underlying optimization problem (3.6).

Let θ now be the *explicitly specified* percentage cost of increasing the capital base. The optimization problem becomes $\max \sum \overline{r}_i V_i - \theta\left[\sum V_i - V_p\right]$ subject to $\Delta V_p^c \geq -CaR_p^{\max}$ (with V_p no longer restricted). This alternative problem resembles situation I where V_p is *not* fixed, but has the same FOCs (3.8) and (3.10) as above. Alternatively, the total financing costs over V_p (not restricted) can be taken into account, leading to $\max \sum\left(\overline{r}_i - \theta\right)V_i$ subject to the CaR constraint. Again this results in the same FOCs (3.8) and (3.10).

3.2 Portfolio optimization allowing for riskfree activities

We now assume that riskfree borrowing and lending opportunities exist for the firm. Denoting activity 1 as riskfree, its return is the riskfree rate r_f. In general, the dollar return on the firm's total activity portfolio is:

$$(3.11)\quad \Delta\tilde{V}_p = V_1 r_f + \sum_{i=2}^{N} V_i \tilde{r}_i = V_1 r_f + \Delta\tilde{V}_q \qquad \text{with} \qquad \Delta\tilde{V}_q = V_q \tilde{r}_q$$

where V_q is the risky part *q* of portfolio *p*, satisfying $V_q \equiv \sum_{i=2}^{N} V_i \tilde{r}_i$.

When V_p is fixed, we have $V_1 = V_p - \sum_{i=2}^{N} V_i \tilde{r}_i$ as a function of the risky ventures. Defining the weight *w* of the risky activities in the total portfolio, the excess total portfolio return is:

$$(3.12)\quad \tilde{r}_p - r_f = w \cdot \left(\tilde{r}_q - r_f\right) \quad \text{with} \qquad w \equiv V_q / V_p$$

Portfolio *p*'s excess dollar return CaR is:

$$(3.13)\quad CaR_{pf} \equiv -V_p\left(r_p^c - r_f\right) = CaR_p + V_p r_f$$

It finally readily follows that the excess dollar return CaRs of p and *q* are equal:

$$(3.14)\quad CaR_{pf} = -V_p\left(r_p^c - r_f\right) = -\left(\frac{1}{w}V_q\right) w\left(r_q^c - r_f\right) \equiv CaR_{qf}$$

We now revisit the four problems in Exhibit 1.

problem I

In this situation the firm strives to maximize the expected dollar return over the unrestricted capital V_p subject to the absolute CaR constraint:

$$(3.15)\quad \Pr\left\{\Delta\tilde{V}_p \leq -CaR_p^{\max}\right\} = \Pr\left\{\Delta\tilde{V}_q \leq -\left(CaR_p^{\max} + V_1 r_f\right)\right\} = 1 - c$$

where we have used (3.11). Hence:

$$(3.16)\quad CaR_q = -\Delta V_q^c = CaR_p^{\max} + V_1 r_f$$

Eq.(3.16) shows that this optimization case is not interesting. The absolute CaR restriction on *p* can be satisfied with *any* risky portfolio *q* simply by adding sufficient riskfree investment $V_1 > 0$.

problems II and III

When considering V_p fixed, the optimization problem is given by eq.(3.6) with FOCs (3.7). For the riskfree activity *i*=1 we have $MCaR_{1p} = -r_f$, so the FOC becomes:

$$(3.17)\quad \theta = (1+\lambda) r_f$$

Substituting in (3.7) yields:

$$(3.18)\quad \overline{r}_i - r_f = \lambda\left(MCaR_{ip*} + r_f\right) \qquad \forall i \in p*$$

Multiplying with V_i and summing over $i \in p*$ gives:

$$(3.19)\quad V_p\left(\overline{r}_{p*} - r_f\right) = \lambda\left(CaR_{p*} + V_p r_f\right)$$

The LHS of (3.19) is the expected excess dollar return (i.e. dollar risk premium) on portfolio *p*, and the term in parentheses on the RHS is *p*'s excess dollar return CaR:

$$(3.20)\quad CaR_{pf} \equiv CaR_p + V_p r_f = -V_p\left(r_p^c - r_f\right)$$

Eqs.(3.18) and (3.19) translate into:

$$(3.21)\quad \frac{\Delta\overline{V}_i^* - r_f V_i^*}{CCaR_{ipf*}} = \frac{\Delta\overline{V}_j^* - r_f V_j^*}{CCaR_{jpf*}} = \frac{\Delta\overline{V}_{p*} - r_f V_p^*}{CaR_{pf*}} \qquad \forall i \in q*$$

with $CaR_{pf*} = CaR_{pf}^{\max}$ in excess return form, provided that $MCaR_{ipf*} \neq 0, \forall i \in p*$. This is the implied portfolio optimality condition, equivalent to:

$$(3.22)\quad \frac{\overline{r}_i - r_f}{MCaR_{ip*} + r_f} = \frac{\overline{r}_j - r_f}{MCaR_{jp*} + r_f} = \frac{\overline{r}_{p*} - r_f}{-r_{p*}^c + r_f} = \frac{\overline{r}_{q*} - r_f}{-r_{q*}^c + r_f} \qquad \forall i \in q*$$

The last equality follows from (3.12) and (3.14).The notable difference with (3.9) is that the denominators are the *excess return* CaR contributions. Dowd [2000, p.221] disqualifies conventional RAROC since it can become infinitely large by investing all capital in riskfree ventures. But we see that the relevant RAROC measure in this limit case becomes indeterminate.

To allow for zero marginal CaRs we rewrite (3.22) as:

$$(3.23)\quad \bar{r}_i - r_f = \frac{MCaR_{ip*} + r_f}{CaR_{p*} + V_{p*} r_f}\left[\bar{r}_{p*} - r_f\right] \qquad \forall i \in p*$$

Multiplying both sides of (3.23) with V_p translates the expression into dollar terms as in (3.21).Note that (3.23) corresponds to (3.4) cast in excess return form.

The FOC eq.(3.22) is identical to the FOC for the mean-VaR portfolio selection problem as derived in Grootveld & Hallerbach [2003]. It reveals linear two-fund separation, i.e. the optimal allocation within the risky portfolio *q* is independent of the total portfolio *p*. For any value of the maximum admissible CaR the corresponding optimal portfolio allocation consists of a linear combination of the risk free investment and only one single risky portfolio *q*.[20] Note that this separation property is fundamentally different from that implied by the maximization of expected utility; for the latter optimization case the results of Cass & Stiglitz [1970] are definitive. The optimal allocation of the available capital is achieved when (3.21) or (3.22) is satisfied, and these conditions apply for portfolio *q*. Given portfolio *q* and the CaR constraint, portfolio *p* readily follows.

The last term of (3.21) (or (3.22) in percentage terms) is the relevant implied risk-adjusted performance metric for this case. The numerator is the (dollar) risk premium on the activity portfolio.[21] The first term of (3.21) or (3.22) shows how to appraise the performance of activity *i*: by relating its risk premium (or average excess return) to its contribution to overall "excess" economic capital $CaR_{pf} = CaR_{qf}$. Again, using a conventional RAROC measure will cloud the performance analysis.

An interesting aspect of this particular problem is that it simply is optimization problem I as studied in section 3.1, but now fully cast in excess returns. Hence the result from the Appendix applies: unconstrained maximization of the implied adjusted RAROC measure (the last term of (3.21) or (3.22)):

$$(3.24)\quad \max\ \frac{\Delta\bar{V}_p - r_f V_p}{CaR_{pf}} = \frac{\bar{r}_p - r_f}{-\left(r_p^c - r_f\right)}$$

yields the same FOCs as in the two LHS terms of (3.21) or (3.22):

$$(3.25)\quad \frac{\Delta\bar{V}_i^* - r_f V_i^*}{CCaR_{ipf*}} = \frac{\Delta\bar{V}_j^* - r_f V_j^*}{CCaR_{jpf*}} \qquad \forall i \in p*$$

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[20] See also Arzac & Bawa [1977] and Tasche [1999] on this point.

[21] Crouhy, Turnbull & Wakeman [1999] *define* an adjusted RAROC measure in the form of RAROC minus the riskfree rate, divided by CAPM beta. This is to account for systematic (priced) risk instead of total firm risk. However, they do not *derive* the metric nor indicate how it could be derived.

Note that the maximand in (3.24) is in spirit similar to the Sharpe ratio[22]: it measures the expected excess return per unit of risk where risk is here defined in terms of the CaR of the excess returns. Without restricting preferences, mean-variance analysis is valid when portfolio returns are elliptically distributed with finite variance.[23] Such elliptical distributions are fully defined by the location and scale parameters and do not exhibit skewness.[24]

In an elliptical world the excess CaR of portfolio *p* is simply defined by the first two statistical moments of the portfolio return distribution:

$$(3.26) \quad CaR_{pf} = -r_{pf}^{c} = -\overline{r}_{pf} + k(c)\cdot\sigma_{pf} \qquad \text{(with } \sigma_{pf} < \infty \text{)}$$

where σ_{pf} is the standard deviation of the excess return on portfolio *p* and $k(c)$ is the proportionality factor belonging to the CaR confidence level c. For example when portfolio returns are normally distributed we have $k(c) = N^{-1}(c)$ where $N(\cdot)$ is the standard normal distribution function. In addition, in an elliptical world CaR satisfies the sub-additivity property; cf. Embrechts et al. [2002]. Incorporating (3.26) in (3.24) and some rearranging yields:

$$(3.27) \quad \max \frac{\overline{r}_p - r_f}{-\overline{r}_{pf} + k(c)\cdot\sigma_{pf}} = \left[-1 + k(c)\frac{\sigma_{pf}}{\overline{r}_p - r_f}\right]^{-1} \Leftrightarrow \max \frac{\overline{r}_p - r_f}{\sigma_{pf}}$$

given *c*. So in an elliptical world, the optimization problem (3.24) entails finding the portfolio *p* that maximizes its Sharpe ratio (the last term in (3.27)). Hence in an elliptical world the two portfolio optimization problems are *completely equivalent*. This contradicts Campbell, Huisman & Koedijk [2001]. However, as we expect that the underlying distributions in a RAPM context are asymmetric, this disqualifies the convenient elliptical parametric assumption.

For convenience, the relevant RAROC measures implied in each of the decision situations is summarized in Exhibit 2. (Recall that IV is indeterminate.)

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22 See Sharpe [1966,1994].

23 See Owen & Rabinovitch [1983].

24 The general class of (both finite and infinite variance) elliptical distributions includes the Student *t* distribution, symmetric stable (Pareto-Lévy) distributions with characteristic exponent smaller than two and the normal distribution. Also non-normal variance mixtures of multivariate normal distributions belong to the elliptical class, see for instance Chmielewski [1981] and Fang, Kotz & Ng [1990].

Exhibit 2: Adjusted RAROC measures implied in each of the decision situations (see in Exhibit 1) of CaR-constrained optimization.

firm's capital	riskfree ventures: no	riskfree ventures: yes ($\exists\, r_f$)
V_p free	**I:** $\dfrac{\Delta\overline{V}_{p*}}{CaR_{p*}}$	**I:** NA
V_p fixed	**II, III:** $\dfrac{\Delta\overline{V}_{p*} - \theta V_{p*}}{CaR_{p*}}$	**II, III:** $\dfrac{\Delta\overline{V}_{p*} - r_f V_p^*}{CaR_{pf*}}$

4. Conclusions

We argue that without explicitizing the relevant EWRM decision context (in the form of clearly and unambiguously stipulating the objectives and constraints), it is *not* feasible to:

- *ex ante* optimize the firm's activities portfolio;
- *ex ante* allocate economic capital over activities comprised in the firm-wide portfolio according to their risk contributions;
- *ex ante* evaluate the quality of the activities portfolio in the light of pursued objectives and imposed constraints;
- *ex post* evaluate overall portfolio performance;
- *ex post* attribute performance to individual activities.

We illustrated our argument with various simple optimization examples. Even though the presented decision contexts are simplified, our results clearly show that applying RAPM on the basis of conventional RAROC measures (as presented in the literature) more often than not may lead to erroneous conclusions and actions.

For deriving relevant RAP measures we suggest the following "recipe". Firstly, identify the relevant objective(s) and constraints. Secondly, derive the implied portfolio optimality conditions from the appropriate optimization problem. Finally, apply the implied relevant RAP measure for *ex ante* portfolio enhancement and *ex post* performance evaluation and attribution. A challenging route for further research is to uncover more complex typological decision contexts as we may encounter them in practice and to reveal the implied – and hence adequate – RAP metrics.

Appendix

Let $A(\boldsymbol{x})$ and $B(\boldsymbol{x})$ be analytic functions of vector $\boldsymbol{x}=[x_i]$, homogeneous of degrees g and h, respectively. Consider the following constrained optimization problem:

$$\text{(A.1)} \quad \max_{\{x\}} \quad A(\boldsymbol{x}) \qquad s.t. \quad B(\boldsymbol{x}) \geq B$$

where B is a (negative) number. The FOCs are:

$$\text{(A.2)} \quad \nabla A(\boldsymbol{x}) + \lambda \nabla B(\boldsymbol{x}) = \mathbf{0}$$

together with the original constraint, where λ is the Lagrange multiplier and ∇ is the gradient operator. We assume second order conditions are satisfied. Premultiplying (A.2) with $\boldsymbol{x}'$, solving for λ and substituting back yields the portfolio optimality condition:

$$\text{(A.3)} \quad \nabla A(\boldsymbol{x}^*) = \frac{\nabla B(\boldsymbol{x}^*)}{B(\boldsymbol{x}^*)^h} A(\boldsymbol{x}^*)^g$$

where asterisks denote the optimum.

Now consider the first unconstrained problem, stipulating a fixed trade-off between $A(\boldsymbol{x})$ and $B(\boldsymbol{x})$ governed by the parameter $\gamma > 0$:

$$\text{(A.4)} \quad \max_{\{x\}} \quad A(\boldsymbol{x}) + \gamma B(\boldsymbol{x})$$

This yields the same FOCs (A.2) and portfolio optimality condition (A.3)

Next consider the second unconstrained problem:

$$\text{(A.4)} \quad \max_{\{x\}} \quad \frac{A(\boldsymbol{x})}{-B(\boldsymbol{x})} \qquad B(\boldsymbol{x}) \neq 0$$

The FOC is:

$$\text{(A.5)} \quad \nabla A(\boldsymbol{x}^*) - \frac{A(\boldsymbol{x}^*)}{B(\boldsymbol{x}^*)} \nabla B(\boldsymbol{x}^*) = \mathbf{0}$$

which translates into (A.3) when $g = h = 1$ (i.e. linear homogeneity).

The third unconstrained problem:

$$\text{(A.6)} \quad \max_{\{x\}} \quad \frac{A(\boldsymbol{x}) - \theta}{-B(\boldsymbol{x})} \qquad B(\boldsymbol{x}) \neq 0, \ \theta \neq 0 \ \text{ constant}$$

has FOCs:

(A.7) $$\nabla A(\boldsymbol{x}^*) - \frac{A(\boldsymbol{x}^*) - \theta}{B(\boldsymbol{x}^*)} \nabla B(\boldsymbol{x}^*) = \mathbf{0}$$

which is incompatible with (A.3), even when $g = h = 1$.

Now let the expected portfolio return $\Delta\overline{V}_p$ correspond to *A(**x**)* above, and the portfolio CaR CaR_p (defined in terms of losses) correspond to -*B(**x**)*. Since $\Delta\overline{V}_p$ and CaR_p are linearly homogeneous in the activities $\{V_i\}_{i \in p}$, maximizing $\Delta\overline{V}_p$ subject to only a constraint on CaR_p is equivalent to maximizing $RAROC_p \equiv \frac{\Delta\overline{V}_p}{CaR_p}$ unconstrained. Moreover, since $RAROC_p$ is homogeneous of degree zero, this measure can be maximized without taking into account any restriction on V_p. The solution can simply be scaled to satisfy this restriction. However, from the third problem (A.6) we have:

(A.8) $$\max \frac{\Delta\overline{V}_p}{CaR_p} \not\Leftrightarrow \max \frac{\Delta\overline{V}_p - \theta V_p}{CaR_p}$$ ■

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