

Risk Capital Aggregation: the Risk Manager's Perspective

Francesco Saita
Associate Professor
Newfin Research Center and IEMIF
Università Bocconi
Viale Isonzo 25
20135 Milano MI
Italy

Tel. +39 02 5836 6102 (dir.)
Tel. +39 02 5836 5908 (secr.)
Fax + 39 02 5836 5921

E-mail: francesco.saita@uni-bocconi.it

First version: November 2003
This version: September 2004

Risk Capital Aggregation: the Risk Manager's Perspective

Abstract

Risk aggregation, defined as the development of “quantitative risk measures that incorporate multiple types or sources of risk” aimed at measuring the overall capital at risk for a financial institution, is a critical topic both for banks and for their regulators. This paper points out the critical role that the choice of the notion of “capital” that is considered at risk may have, therefore discussing the issue of business risk, that has so far received very little attention in the literature. The paper then discusses alternative risk aggregation techniques and some of the problems that arise in the estimation of their parameters. Parameter estimation appears to be a major concern when deciding which aggregation technique to adopt, especially considering the implications for risk-adjusted performance measurement and therefore for decisional processes that may derive from the risk aggregation exercise.

Risk Capital Aggregation: the Risk Manager's Perspective^{*}

1. Introduction

Defining the optimal level of capital and its best possible allocation across businesses is important for any kind of firm. In the financial sector, this problem has gained increasing attention and has attracted a substantial amount of investments by banks and other financial institutions due to the parallel and intertwined development of regulatory capital requirements and of risk measurement techniques (typically based on the concept of Value at Risk). At present, most if not all large banking groups have specific models for evaluating market and credit risk for at least a large part of the legal entities they comprise, and are investing in models of increasing sophistication to assess and quantify operational risk. In the past few years, their efforts centered on credit risk models and internal rating systems, partly as a consequence of the crucial phase of the New Capital Accord. However, the issue of risk aggregation, i.e. the development of “quantitative risk measures that incorporate multiple types or sources of risk”¹, is now recognized as critical both for banks and for regulators. In fact, if a risk manager were able to measure individual risks on a stand-alone basis only, it would be very hard for him to address the two key questions about (i) whether the bank has the “right” amount of capital and (ii) how to allocate capital across different business units and different risks. Both issues require to be able to determine an integrated measure of the economic capital required by the bank in order to cover potential losses. Anyway, “it is clear

^{*} I am grateful for very useful comments to Luca Erzegovesi, Mario Masini, Davide Maspero, Paolo Mottura, Ramon Rabinovitch, Andrea Sironi, and to participants at the 2004 EFMA Conference in Basle, and in particular to Amrit Judge. The usual disclaimer applies.

¹ See Joint Forum Working Group (2003). The Working Group makes a distinction between risk *integration* (i.e., developing a common risk measurement framework in terms of policies and procedures, tasks and responsibilities, systems, and so on) and risk *aggregation*.

that risk aggregation and economic capital methods are still in early stages of evolution”, as clearly stated in the recent Joint Forum Working Group (2003) survey on *Trends on risk integration and aggregation*.

The topic of risk aggregation is therefore attracting an increasing interest from researchers. Matten (1996) has been the first one to address the issue of deriving an overall measure of capital at risk combining different kinds of risks, adopting a pragmatic perspective from the bank’s internal viewpoint². More recent papers have instead either reviewed banks’ best practices concerning risk aggregation (Joint Forum Working Group 2003) or proposed aggregation techniques to be adopted (see Alexander and Pezier 2003, Dimakos and Aas 2003, Pezier 2003). Of course, considering the pitfalls of common dependence measures such as linear correlation when aggregating return distributions – see in particular Embrechts, McNeil and Straumann (1999) – there is a lot of interest about the role of copulas in risk aggregation. Recently, Rosenberg and Schuermann (2004) have developed an empirical test on the aggregation of market, credit and operational risk with a detailed sensitivity analysis of the impact of different copula functions and correlation parameters on aggregate VaR estimates. The same kind of methodology has been discussed and used by actuaries with reference to risk aggregation in the insurance business or among conglomerates: see in particular Wang (2002), Ward and Lee (2002), Venter (2003). Some contributions have also directly addressed the problem from regulators’ point of view concerning whether and how diversification benefits among businesses or risks may and should be taken into account in setting minimum capital requirements (see Kuritzkes, Schuermann and Weiner 2001 and 2002, and again, Joint Forum Working Group 2003).

² The related issue concerning “diversified” or “undiversified” measures of Value at Risk for individual business units has been discussed in the same period also in Merton and Perold (1993), James (1996), Saita (1999), Culp (2000). Froot and Stein (1998) and Perold (1999) have addressed the topic of capital allocation with a more

Considering both the technical hurdles that need to be overcome in order to obtain an aggregated risk measure and the relevance of the topic from a regulatory viewpoint, the focus adopted by most of the contributions about risk aggregation is completely understandable. Yet, the discussion has left in the background some problems that may be extremely relevant from the risk manager's viewpoint. The aim of this paper is to give a contribution precisely in this direction, trying to give a useful complement to the existing literature in the field.

In particular, the main contributions of the paper are the following. First, while the issue of risk integration and aggregation is related typically to market, credit and operational risk, this paper will discuss the role of business risk, defined here as the risk associated to earnings volatility not determined by event-driven operational risks. We will discuss the key issues in measuring earnings volatility and propose new methods to translate earnings volatility into a measure of capital at risk. Second, the paper will show that the aggregation of capital at risk measure may be analyzed in the context of different notions of "capital". In particular, there is a difference between the book value of capital and the market capitalization that may be relevant especially when addressing the role of business risk. While regulators may correctly be worried mainly about the risks that might cause a bank to fail, the risk manager should be concerned also about risks that while generating moderate losses in the short term (and therefore with little impact on the bank's short-term likelihood to face a distress) may impact substantially on its forecasted earnings and therefore on its market capitalization, destroying value for the shareholders. Third, the paper will discuss the different purposes for which an integrated risk measure may be necessary, and hence which criteria the "best" measure should ideally satisfy. Since such aggregated risk measures should impact on real decision making processes and may enter into business units' evaluation, precision – while remaining crucial –

formal approach, and have clarified the potential impact of economic capital and risk-adjusted performance measures on capital management and capital allocation decisions.

may not be the only desirable attribute. Alternative aggregation techniques will be analyzed in front of such criteria. Fourth, the paper will explicitly address calibration issues in the debate concerning the “optimal” risk aggregation technique, pointing out the risks inherent in using short term (e.g., monthly) earnings data to calibrate the models.

Two brief premises are needed. First, this paper does not support the idea that a single number of aggregate risk capital at risk can be scientifically derived, and magically solve any problem for a CEO that has to take a decision on a bank’s capital structure. Estimates of risk capital may be difficult even at single risk level and become much more uncertain at bankwide level; and anyway, even if a perfect single number could be obtained, capital management choices would never be mechanical. Therefore, while it is important to try to increase the quality of overall risk estimates, overconfidence in the final number(s) a risk aggregation technique may produce remains extremely dangerous.

Second, even if throughout the paper we will mainly consider value at risk (or equivalently capital at risk) as the typical risk metric, we do not claim that it should be viewed as the best or only risk measure. Starting from the seminal paper of Artzner, Eben, Delbaen and Heath (1999), Value at Risk, defined as the maximum amount that may be lost over a certain time horizon within a given confidence interval, has been increasingly criticized since it does not fulfill all desirable attributes a coherent risk metric should possess.

In particular, Value at Risk is not subadditive, i.e. given two portfolios X and Y it is not always true that $VaR(X+Y) \leq VaR(X) + VaR(Y)$. Consequently, alternative coherent measures have been proposed, with particular reference to expected shortfall (ES) that can be defined as the average loss over a certain time horizon within a given portion of the left tail of a portfolio’s return distribution. Expected shortfall is subadditive, and this is clearly a highly desirable quality in the context of risk aggregation. In the paper we will mainly refer to capital

at risk for the reason that, despite these critiques, capital at risk remains at present the typical risk metrics adopted in almost all banks. Yet, many of the comments that we will make could also be applied to other alternative risk metrics.

The paper is organized as follows. Section 2 analyses the different aims of an aggregated risk measure, explains the problem of alternative notions of “capital” at risk that may be adopted, and discusses the criteria that may be used in evaluating aggregation techniques. Section 3 is devoted to business risk, analyzing how a capital at risk measure may be derived depending on whether it is evaluated in terms of book value of capital or market capitalization. Section 4 describes alternative risk aggregation techniques and the issues related to parameters’ estimation. Section 5 concludes.

2. Risk aggregation from the risk manager’s viewpoint

2.1. The possible aims of an aggregated risk measure

Why is an aggregated risk measure useful in a bank? A first clear objective is to assess whether available capital is adequate to cover the bank’s risks. An aggregate measure of capital at risk may therefore support capital management and capital structure decisions (even if one might then question whether a “normal” or an “extreme” aggregate capital at risk measure is warranted)³. Second, it may be used in order to measure the overall risk of the bank so to then decompose it into each business unit’s contribution, defining a “diversified” risk measure as a basis for (a) evaluating business areas so to support top managers’ decisions and (b) evaluating business units and maybe defining the bonuses for those who run them. Third, it may be used to assess marginal contributions of business units or even individual deals to the overall riskiness of the bank, so to define risk pricing rules which may be consistent with the marginal risk associated to any transaction. Keeping in mind these

different purposes is relevant since many authors note that the choice of the risk metric and of the aggregation technique should be purpose-specific (see Venter 2003, Pezier 2003).

The complexity of the different aims that may be associated to risk aggregation is relevant for two implications, that are typically not discussed in existing literature on risk aggregation. The first is that one should question *which* capital has to be aggregated, since different notions of capital may be used and are relevant depending on the purpose of the risk manager. The second one is that risk aggregation in the real world is not a purely technical exercise, since it may impact outsiders' perceptions of the bank's risk, internal allocation of capital and other resources, individual business unit leaders' evaluation and compensation. Worries about these topics may (and should) be considered by the risk manager when choosing an aggregation technique.

2.2. Defining the criteria to evaluate risk aggregation techniques

Considering the different possible objectives of an aggregate risk estimate, there are different criteria that may be considered when evaluating alternative solutions. Some of them are objective and structural (i.e, the theoretical soundness of the model, the intrinsic difficulty in parameters' calibration), others are still objective but may be temporary and transient (e.g. the ability to reduce sampling error or model risk when parameters are estimated without adequate time series of returns), while others are subjective since they depend on the purposes for which the risk manager will use the aggregation technique.

For instance, attributes such as transparency, perceived fairness, or the ability to produce diversified CaR measures in an easy and consistent manner will matter more or less depending on whether the risk aggregation exercise is intended to support only top management's decisions on capital structure, or instead also to guide the internal capital

³ See Pezier (2003).

allocation process, or even to impact (through measures of “diversified” capital at risk) business units’ leaders evaluation and compensation. This also means that choices of different techniques made by different banks might find a rationale in the different breadth of purposes to be pursued.

Some of the criteria may need to be analyzed more closely. While the theoretical attributes and the ease of calibration may be relatively clearer, and will be discussed in Section 4 in more detail, the sensitivity to sampling error is an issue since in most cases the time series over which to try to estimate the parameters (whether they are simple linear correlations or sophisticated copula parameters) may be short or extremely short⁴. Adding new data on a short series could lead the risk manager to change correlation parameters’ estimates, therefore making the aggregated risk measure unstable in early periods even in absence of real market condition changes. Potential “jumps” of the aggregate risk measure would be difficult to explain for the risk manager to the board and to top managers and – if the risk aggregation technique were used also to produce measures of diversified capital at risk for individual business units – may cause discontent and face greater opposition by business units’ managers, especially by those who feel to be negatively affected by the new technique to allocate diversification benefits⁵. A risk manager may then favour at least for the first few years a technique which is less sophisticated but appears to produce results that are more stable as the sample for correlation estimates changes, rather than resorting to a theoretically superior approach that may cause higher instability of estimates.

⁴ Rosenberg and Schuermann (2004) adopt for instance in their interesting simulation on risk aggregation a method in order to reproduce a longer time series of quarterly returns from market and credit risk than they originally have by analyzing the relationship between market and credit risk returns and certain macro factors, for which they have a longer time series. While suggesting an interesting solution, their choice also pinpoints the problems concerning the length of available data series.

⁵ On this topic see also Hall (2002). Of course this political problem would not exist if division managers were evaluated on a stand-alone, undiversified measure of the capital at risk of their unit, without considering diversification benefits.

Transparency and perceived fairness again are relevant if the measures that are produced also have as a byproduct risk measures for individual business units. It is difficult to motivate people to pursue challenging RAROC or EVA targets if they feel that a large part of the result depends on how diversification benefits are allocated to business units based on some mysterious formula whose parameters are secretly handled by the risk management unit. Again, this may lead to favour simple techniques until the more sophisticated ones have been clearly accepted by most managers.

2.3. The notion of “capital” at risk and the role of business risk

The first key decision that an aggregate risk measure should support concerns the optimal amount of capital that a bank should hold so to be adequately protected against the risks deriving from its activity. One clear condition the bank should satisfy is that his regulatory capital should be greater than its minimum regulatory requirement. At the same time, a bank could produce an internal estimate of capital at risk and compare it with its available capital. The internal estimate might consider also on one hand the risks that are not linked to a pillar one capital requirement directly, and on the other hand the diversification benefits that are not considered at present in defining minimum regulatory capital charges, due to the adoption of a building block approach⁶. The notion of “available” capital may vary, since in order to run an internal evaluation a bank may decide to adopt a notion of capital that does not contain, for instance, all Tier 2 and Tier 3 components of regulatory capital. The typical problem in this case is to check the ability of the bank to withstand severe adverse conditions without failing; the choice of a book value definition of available capital and of a one-year horizon as the common denominator to estimate aggregate losses seems reasonable given the objective.

At the same time, yet, in order to defend shareholders' interests, the risk manager may be willing to measure capital at risk also in terms of market capitalization of the bank. Adopting this view would imply a completely different judgement about the risk of certain businesses, since there are some that may generate small short term losses but have a much greater impact on long term earnings expectations by analysts and investors, and may therefore influence market cap significantly. Consider for instance the case of an asset management firm inside a banking group. If we consider the impact the business may have in terms of potential losses of book capital over a one-year horizon, its risk is low, since direct losses would be incurred only if the total fees paid by the customers were unable to cover its variable and fixed costs. At the same time, yet, if the asset management company faced a sharp reduction of management and other fees analysts may revise downwards earnings' expectations for the banking group in the future, and this may have a much more visible effect on target prices and eventually on stock market capitalization of the bank holding company. This may be true for many other fee-generating businesses.

Of course, we are not saying that regulators should be concerned with "market capitalization at risk"; systemic risk is linked to the possibility that a bank may fail, and from this viewpoint a business that may give a very small contribution to a bank's failure should be considered as a low risk business. It is therefore perfectly correct that fee-generating business be associated at present only to an operational risk capital requirement, with no specific requisite for intrinsic business risk. Instead, from the shareholder's perspective and hence ideally also from bank top management's viewpoint a sharp reduction in market cap caused by a fee-based business is clearly bad news even if it is not associated to any change in the bank's default probability. Top managers should at the same time try to prevent the bank from failing and try

⁶ Discussing the relationship that should exist between diversification benefits and minimum capital requirements is beyond the scope of this paper. The reader may refer to Kuritzkes, Schuermann and Weiner

to increase the value of the firm in the market *while controlling the volatility of this value*. Therefore having an idea of how great a portion of market capitalization may be at risk in case of adverse events (different from operational risk ones) inside a single business may be useful information for them.

As a consequence, an interesting question is which notion of capital should be used when calculating risk-adjusted performance measures, such as RAROC and EVATM, for the bank's business units⁷. Of course, the contribution of fee-based business units will be remarkably different depending on which kind of capital is used. Even if both a measure based on book value capital at risk and measure base on market capitalization at risk may be useful, probably the latter is more adequate in providing a support to top managers' decisions for the allocation of capital and other critical resources if they want to make the value of the firm greater and at the same time less volatile. In a shareholder's perspective, then, measuring the contribution of business risk may be much more important than it is in the light of guaranteeing systemic stability: this is why in section 3 we will discuss how business risk can be measured and how it can be translated into a measure of capital at risk, depending on which notion of "capital" is adopted.

3. Measuring business risk

The starting point in any effort to define an integrated measure of capital at risk is the identification of all relevant risks and the definition of a model to quantify their impact under a common metrics. In practice, this metrics is largely identified by capital at risk or Value at

(2001, 2002), and Joint Forum Working Group (2003).

⁷ RAROC (risk-adjusted return on capital) is intended here as a generic risk-adjusted performance measure equal to the ratio between the profit, or margin, produced by a business unit and the capital at risk associated to the same unit. EVATM (economic value added), in the form in which it is typically applied to banks, is intended as the difference between the profit or margin of the unit and the cost of capital, calculated as the product of its

Risk. In the past years, methodologies for market and credit VaR have been developed and improved so to become extremely widespread, while loss databases and measurement methodologies for operational risk are currently under development. Business risk, instead, has received so far less attention, also due to the fact that there is no minimum capital requirement linked to it. Yet, the volatility of revenues deriving from fee-generating business such as asset management and brokerage may have a substantial impact on a bank's P&L, and may influence equity analysts' valuations and the bank's market capitalization. From the risk manager's point of view, independently from the absence of any regulatory constraint, quantifying this kind of risk is therefore an important task.

In order to measure business risk, a bank can in theory adopt one of the following solutions⁸.

- (a) Analyze the average capital of a sample of mono-business competitors and set capital at risk for the fee-generating business unit equal to some simple or weighted average of that sample, using, therefore, a measure of *benchmark capital*.
- (b) Identify risk with a measure of *earnings at risk*, e.g. a certain multiple of the volatility of earnings of the fee-generating business unit⁹.
- (c) Calculate earnings at risk as in case (b) and then translate this measure in some way into a measure of *capital at risk*.

The first solution is conceptually simple but faces several problems. First, it may not be possible to find pure mono-business competitors. Second, even if they exist, it may be necessary to scale up or down their capital - that may cover not only business but also other

capital at risk multiplied by a target return on capital. For a more detailed analysis of risk-adjusted performance measurement see James (1996), Matten (1996), Saita (1999), Culp (2000).

⁸See Matten (1996). The Joint Forum Working Group (2003) survey reports that only some among the banks that have been contacted are really measuring in some way business risk integrating it in the overall risk management framework.

⁹See again Matten (1996) for a more detailed description. Depending on data availability, in theory one could also decide to derive analytically the desired percentile on the historical distribution of earnings, but typically data are insufficient. In other cases, due to the lack of proper data with a reasonable frequency, a measure of

risks, such as operational risk - according to differences in size between the internal business unit and the set of peers that has been selected. Third, their level of capitalization may be influenced by minimum regulatory requirements that may be binding for a mono-business competitor while this would not happen if the same activity were conducted within a diversified group. Finally, since the measure depends on external data only, there is no link between the behaviour of the business unit and the benchmark capital at risk it is assigned.

These problems can be overcome adopting approaches (b) or (c). Since determining an Earnings at Risk measure is a prerequisite for both, we will first discuss how such a measure can be derived and then its possible translation into a capital at risk measure.

3.1. Earnings at Risk estimation

In theory, earnings at risk (EaR) can be identified with the losses generated by a given business unit under the maximum adverse variation in its earnings within a given time horizon and confidence interval. For instance, for the asset management business, one could try to build a distribution of the net margin produced by the fund management company and identify EaR at a 99% confidence level as the 0.01 quantile of the margin distribution.

In practice, yet, defining business risk properly is the first task for the risk manager. In particular, the risk manager should decide whether to identify Earnings at risk with the deviation from expected earnings or only with actual losses. Imagine for instance that for the business unit Alpha expected earnings were equal to 50 and the maximum adverse reduction in earnings at a 99% confidence level is equal to 120: Earnings at Risk should then be 120 (the maximum potential reduction in earnings) or just 70 (the actual loss in the worst case scenario)? Using actual losses has the disadvantage that the measure may become very

volatility of earnings can be derived by measuring the volatility of revenues and then introducing careful assumptions about the structure of variable versus fixed costs for the business unit.

sensitive to changes in the internal cost allocation: a bank that allocates to final business units a higher share of its indirect costs would measure – everything else being equal – higher potential actual losses for each business unit than an identical bank that allocates only direct costs. This may both impact evaluation of different business units and overall business risk estimates (in this case, only provided that there is no correction at aggregate level for the share of unallocated costs).

After defining whether or not to consider expected earnings, the risk manager has to derive that measure resorting typically to a time series of a business unit's reported earnings. As in the case of operational risk, data availability may be a major issue.

First, the method cannot be applied at all when there is no time series of earnings available. This happens for instance when business units have been restructured, or merged, and the structure of the information system containing accounting data is not flexible enough to reconstruct a theoretical time series for the “new” business units¹⁰.

Second, the time series of earnings is likely to include the effect of some adverse events that may be attributed to operational risk. It would therefore be possible to consider the same source of risk twice, once by including the event in the operational risk database and the second time by considering its adverse effect on earnings volatility that leads to business risk estimation. Caution is therefore necessary to derive a time series that filters out the effects that are linked to risks that are separately measured.

Third, the risk manager has to build a series of returns from earnings data, which implies the definition of a proper measure of “exposure” to business risk. There are two reasons why returns instead of pure earnings is needed. If for instance the size of the business unit had substantially changed through time, then higher or lower earnings in certain periods may reflect a size change rather than earnings volatility. At the same time, when estimating

correlation across business units' business risk, the dependence across business risk in different units should be measured through returns, that do not depend – differently from earnings – on the relative size of different business units. Transforming earnings into returns has yet the shortcoming that it is necessary to identify a size, or exposure, measure, that should be proportionally linked with returns. This measure should also be used at the end of the process in order to translate a percentage EaR measure into a dollar EaR value. Yet, the choice of such scaling factor may be often judgemental and arbitrary. This explains why risk managers may be tempted to use dollar earnings' distributions in order to estimate earnings at risk directly from these series.

Fourth, it is difficult to select the right time period from which to extract data to estimate earnings volatility. If the period is too long, then the measure may depend on old data that may no longer be representative of the volatility of the business unit. If, on the contrary, one decides to consider a relatively short time interval, than it is almost a must to use monthly earnings data in order to have a sample of a minimum size. This in turn has two shortcomings. On one hand, with such a small number of data, it is very difficult to estimate the exact shape of the distribution of earnings empirically; hence, the identification of the desired distribution percentile used to derive an EaR measure cannot be based on a sound ground (and many may end up assuming that the earnings distribution of any business unit is – magically – normally distributed). On the other hand, using monthly data can lead to an incorrect estimate of earnings volatility if – as it may easily happen – there is some serial positive autocorrelation in earnings changes, or if the way in which accounting earnings are calculated tends to alter in some way the volatility of the “real” results of the business units¹¹. Yet, since it is not always

¹⁰ This situation is typical in the case of banks which have recently experienced a merger.

¹¹ This phenomenon may not depend on a precise will to influence financial results, but it may simply derive from the way in which costs are attributed (e.g. being equally split in each month even if they are partially variable).

possible to scale monthly return volatility into annual volatility in a simple manner, the annual earnings at risk figure may be derived by numerically simulating a series of monthly shocks, and autocorrelation – in case it exists – in monthly returns could be simply taken into account through the same simulation procedure.

3.2. From Earnings to Capital at Risk

After estimating an EaR measure, the other problem is whether such a measure can be properly compared with measures of *capital* at risk. Matten (1996) observes that the EaR measure is typically disproportionate to capital at risk measures for risks such as market or credit risk, and this would imply the attribution of an equally disproportionate RAROC¹² to fee-generating business units. He therefore suggests to translate earnings at risk into capital at risk by dividing EaR by the risk-free rate: capital at risk would therefore be conceived as the amount of capital that should be invested at the risk-free rate in order to cover the loss coming from the fee-generating business unit. Therefore capital at risk (CaR) associated to the business unit is defined so that

$$CaR \cdot r = EaR \quad (1)$$

and hence

$$CaR = \frac{EaR}{r} \quad (2)$$

where r is the risk-free rate. This method is simple and requires only one input: the risk-free rate. However, the amount of capital that is obtained in this manner is not capital that is lost forever (as it would happen, for instance, for a market risk CaR in case of an adverse move of the markets) but only an amount of capital for which the investor faces an opportunity cost.

¹² We define here RAROC (risk-adjusted return on capital) in general terms as a ratio between a measure of earnings of a given business unit (or business area, or even transaction) and the capital at risk associated to it; we consider therefore RAROC as the classic risk-adjusted performance measure.

As an alternative to this simplified method, we therefore propose here some new methods to translate the EaR measure into a CaR measure, and that can be derived by analyzing the impact on the economic value of the bank of a loss equal to EaR. The key idea is to adopt an equity analyst's point of view as if we wanted to evaluate how the reduction in earnings would affect the target price and therefore the theoretical economic value of the bank. In this case, there are different possible solutions depending on the criteria that the analyst is assumed to follow.

For instance, let us assume that the analyst simply evaluates the value of the equity stake in the bank through a multiple, such as the price/earnings ratio (P/E). Then, the adverse change in the market capitalization of the bank (i.e., the change of economic capital in the worst case scenario, DEC_{wcs}) that would derive from an unpredicted reduction in earnings would be equal to

$$CaR_x = \Delta EC_{wcs,x} = \left[\frac{P}{E} \right]_x^* \Delta Earnings_{wcs} = \left[\frac{P}{E} \right]_x^* EaR \quad (3)$$

where the term

$$\left[\frac{P}{E} \right]_x^*$$

denotes the fair price/earnings ratio that the analyst attributes to business unit x . This term could therefore vary across business units, since they can be valued by analysts at different multiples.

A similar logic could be followed if analysts were assumed to adopt a dividend discount model. In the simple case of a constant growth dividend discount model, defining k_e the discount rate, g the estimated dividends' perpetual growth rate, and D the next dividend per share, the price P of the share should be equal to

$$P = \frac{D}{k_e - g} \quad (4)$$

so that expressing the dividend D as the earnings per share multiplied by the payout ratio, p , and multiplying both terms by the number of outstanding shares, the market capitalization EC is equal to

$$EC = \frac{p \cdot Earnings}{k_e - g} \quad (5)$$

If we assume that the total economic value of the bank is evaluated in this way by the market, then the reduction in the economic capital of the bank that would follow a reduction in the value of earnings equal to EaR would be

$$CaR_x = \Delta EC_{wcs,x} = \frac{p \cdot \Delta Earnings_{wcs,x}}{k_e - g} = \frac{p}{k_e - g} \cdot EaR_x \quad (6)$$

In this case¹³, the Capital at Risk estimate could even be too conservative, since it implicitly assumes that the unexpected reduction in earnings represented by EaR would negatively affect the level of future earnings of each year in the future (since they will be calculated capitalizing earnings in year 1 at the rate g). However, reduction in earnings are sometimes combined with a downward revision of the growth expectations for certain areas of business. Thus, at least for certain business units a cautious risk manager could question whether the potential adverse variability of g should also be taken into consideration.

Translating the impact of earnings volatility into capital at risk adopting the analyst's view can also lead, under certain simplifying assumptions, to a solution that is reasonably close to Matten's (1996) proposal. To see this, let us assume the analyst evaluates the total value of the bank as the sum of discounted cash flows, so that the value of the bank V is

¹³ If one accepts the assumptions of the constant growth dividend discount model, then the term $p/(k_e - g)$ in equation (6) is the theoretical value of the "correct" P/E, so that equation (6) is simply a different formulation of equation (3), suggesting an alternative way to estimate the correct multiple when a direct estimate is not feasible.

$$V = \sum_{t=1}^{\infty} \frac{F_t}{(1+i)^t} \quad (7)$$

where i is the proper discount rate. Since a change in V would affect directly the shareholders' wealth, CaR is equal to ΔV . If we assume also that (a) a reduction in earnings equal to EaR_x in business unit x would imply an equal reduction in cash flows and (b) this reduction is perpetual, then labeling V_0 the value before the shock in earnings and V_{wcs} the post-shock value, capital at risk is equal to

$$\begin{aligned} CaR_x &= V_0 - V_{wcs} = \sum_{t=1}^{\infty} \frac{F_t}{(1+i)^t} - \left(\sum_{t=1}^{\infty} \frac{F_t - EaR_x}{(1+i)^t} \right) = \\ &= \sum_{t=1}^{\infty} \frac{EaR_x}{(1+i)^t} = EaR_x \cdot \sum_{t=1}^{\infty} \frac{1}{(1+i)^t} = \frac{EaR_x}{i} \end{aligned} \quad (8)$$

This solution is close to the one suggested by Matten (1996) but (a) it has a completely different derivation and (b) it uses the discount rate i rather than the risk-free rate (and therefore produces a lower equivalent capital at risk, since $i > r$). This also clarifies why Matten's proposal, although clearly going in the right direction, could be considered too pessimistic: in fact, it assumes that capital at risk is even higher than the loss in economic value that the bank would face if the loss of cash flows were *perpetual*. Of course, by assuming equal changes in cash flows and earnings¹⁴ and that a loss equal to EaR would persist for n years (instead of being perpetual) the estimated capital at risk would be reduced to

$$\begin{aligned} CaR &= V_0 - V_{wcs} = \sum_{t=1}^{\infty} \frac{F_t}{(1+i)^t} - \left(\sum_{t=1}^n \frac{F_t - EaR}{(1+i)^t} + \sum_{t=n+1}^{\infty} \frac{F_t}{(1+i)^t} \right) = \\ &= \sum_{t=1}^n \frac{F_t}{(1+i)^t} - \sum_{t=1}^n \frac{F_t - EaR}{(1+i)^t} = \sum_{t=1}^n \frac{EaR}{(1+i)^t} = EaR \cdot \frac{1 - \frac{1}{(1+i)^n}}{i} \end{aligned} \quad (9)$$

¹⁴ Of course, if a bank wanted to adopt this method, a direct estimate of cash flows variability would be advisable.

The assumption whether a loss equal to EaR today would be considered by analysts as transient and recoverable through time or, instead, as a sign of permanently lower earnings in the future becomes therefore critical¹⁵. In general, the alternative solutions proposed here require more inputs than Matten's method, and may lead to different CaR values starting from similar EaR measures, because different business units may have different price/earnings multiples or different customer relationships' average duration. Therefore, they may be more easily questioned by the business units under scrutiny, which might also try to negotiate the parameters with the risk management function. In any case, they have a sound rationale and can be made compatible with any possible evaluation criterion that banks' equity analysts favour in a given context.

As a final but very important remark, the translation of EaR into CaR implicitly assumes that the bank wants to measure its capital at risk in terms of the reduction in its market capitalization due to an adverse event: in fact, we have identified CaR through the impact of potential losses on the target price of the bank's stock. This point will be discussed in more detail in the next paragraph.

3.3. Harmonizing capital at risk measures

We showed how a measure of capital at risk can be derived for all different risks, but then the question still remains of how homogeneous those measures can be. As it is well known – see also Joint Forum Working Group (2003) – there are at least two choices that require harmonization: the time horizon used for the CaR estimate, which is usually daily for market risk and yearly for credit risk, and the confidence interval, where 99% is the standard for market risk, whereas in the case of credit risk and at a corporate level a higher rate such as

¹⁵ This could be based on empirical data concerning the average duration of a customer relationship, if adequate data are available.

99.97%, consistent with the target rating of the bank, is generally preferred. While these effects are quite well-known, the third issue related to the different concept of “capital” underlying the measures of capital at risk is typically neglected.

The harmonization of time horizon is normally achieved by adopting a one-year horizon for all risks and by scaling up market capital at risk by multiplying it by the number of trading days in a year¹⁶. Alternatively, when a risk manager does not need to calculate the annual equivalent of a single value (e.g., market CaR calculated on December 31st) but rather the annual equivalent of the market risk the bank has faced over a certain period of time (based on a string of daily CaR values), one could calculate annual CaR as the square root of the sum of n daily CaR values in each day throughout the year according to the formula¹⁷

$$CaR_{annualized} = \sqrt{\sum_{i=1}^n CaR_{daily,i}^2} \quad (10)$$

The harmonization of confidence interval is sometimes obtained by rescaling the 99% VaR measured for market risk to the desired and typically higher percentile using a fixed scaling factor *as if* the distribution were Gaussian. Needless to say, this can be false in practice, but a direct estimate at the 99.97% confidence level may be very unstable and questionable, e.g. if the bank uses an historical simulation method which would require interpolation to determine

¹⁶ See for instance Joint Forum (2003).

¹⁷ See Saita (1999). The formula is derived considering annual CaR as a multiple of the volatility of yearly profit and losses, conceived as a random function equal to the sum of n independent random functions represented by daily profit and losses. The formula therefore replicates the relation between the standard deviation of a variable Y which is the sum of n independent random variables X_1, X_2, \dots, X_n .

It can be noted that this solution is different from annualizing daily CaR values and then taking their average as the annualized CaR for, say, the whole year. In fact, this latter and apparently simpler solution would not properly take into account the risk related to peak exposures. Consider a period of 25 trading days and two traders. Let us assume that trader A had a daily CaR equal to 10 during each trading day, while trader B had no risk (and zero CaR) for 24 trading days and a CaR equal to 250 in one trading day only. Even if both have an average daily CaR equal to 10, it is evident that the second generates a higher risk of failure for the bank: in fact, his trades are not diversified through time, and there would be no chance for the bank to intervene and stop trading after a certain amount of losses as it would happen for the first trader. While by annualizing daily CaR values and then averaging them one would obtain the same equivalent CaR estimate for both traders, by using equation (10) trader B would properly be assigned a much higher period-equivalent CaR than his fellow. In fact,

that exact percentile. Moreover, since different business units would have different mixtures of different risks that have in turn different shapes of return distributions, the choice of the confidence level is not neutral with respect to the relative capital at risk absorption of different business units (see Hall 2002), and therefore it may influence not only the absolute level of reported RAROC for a unit, but also its relative ranking inside the organization. This is why defining the confidence interval in an objective manner (e.g. linking it to the target rating the bank is willing to maintain) is very important.

The third relevant issue in harmonizing CaR measures is related to the notion of “capital” that underlies capital at risk measures for different kind of risks. There are at least three different versions of capital that can be and are used:

1. Market capitalization value of capital (i.e. the value of the equity of the bank on the stock market).
2. The market value of capital (i.e. the difference between the value of marked-to-market assets and the value of marked-to-market liabilities, without including any form of goodwill or badwill).
3. The book value of capital (i.e. the difference between the value of assets and of liabilities at book values).

Trying to simplify and reduce the possible behaviour of a bank to a few most common alternatives, we can say that:

- market CaR is measured in mark-to-market terms;
- credit CaR may be measured either at book values (especially as far as loans are concerned) or at mark-to-market values, or both;

the overall CaR measure for the whole period according to equation (10) would be equal to 250 for trader B and to $10\sqrt{25}$ only for trader A.

- business risk CaR can be measured either at book values (e.g. as a measure of one-year Earnings at Risk) or as the market cap value of capital (by applying some of the methods described in the previous paragraph to convert an EaR into a CaR measure).

This lack of consistency between capital at risk measures could be solved differently depending on the objective. If the goal is to have a consistent measure in order to calculate risk-adjusted performance (RAP) measures to support capital allocation, then market cap value of capital is likely to be the correct solution since it is the most consistent with the shareholder's view of the bank. If one wants to check whether the bank is sufficiently capitalized or not, in order to support decisions about the bank's optimal capital structure policy, then it can be argued that in theory one should like CaR measures not only to be intrinsically homogeneous, but also to be consistent with the measure of available capital with which they are compared. It is therefore possible to compare for instance a book value measure of capital at risk and compare it to available capital at book values, and an economic measure of capital at risk and compare it to capital measured as the current economic capital of the bank at stock market prices. The same may happen at mark-to-market prices, so to derive a set of co-existing constraints for the bank's survival, (having of course the fourth key boundary of maintaining regulatory capital higher than minimum capital requirements). This approach differs from the common practice to assume that all – albeit different – measures of capital at risk have to be compared against a single measure of available capital. In practice, it may clearly be hard and costly to maintain different measures of capital at risk according to different criteria, and try to correctly explain the differences to the board members that have to take key decisions. Yet, it is important to try to attain consistency of the CaR measures that are used; moreover, accepting this view some problems become easier to be managed. For instance, while for business risk the risk manager apparently has to choose between a pure

EaR measure or a much higher CaR measure¹⁸, if different notions of available capital are considered then the two measures become an “accounting” capital at risk measure and a “market cap” capital at risk measure that must be compared with different amounts of available capital (the larger CaR measure being compared with the higher amount of capital).

4. Firm-wide integration and the issue of correlation across risks and business units.

Aggregating different risk measures into a single number for the whole institution requires to tackle at least three different issues: (a) identifying the components that have to be aggregated, (b) identifying the aggregation technique or algorithm to be used, and (c) calibrating the parameters (e.g. correlation coefficients) needed to derive the single risk measure. We will approach these issues precisely in this sequence.

4.1. The choice of the components to be aggregated: business units versus risk types

Given a homogeneous measure for all risks, the problem is how to aggregate individual risks to obtain a single capital at risk measure. The first issue is the choice between business units and risk types as the starting point for risk aggregation¹⁹.

Aggregating across business units has the advantage of a clear link between the individual divisions’ CaR measures and the overall capital requirement of the bank. Moreover, correlation coefficients can be estimated through the series of the P&Ls of the business units. At the same time, there are two clear disadvantages. First, since a single business unit is typically exposed to multiple risks, the need to define or assume a correlation coefficient across risks is not avoided at least when calculating stand-alone CaR for the individual business unit. Secondly, if the same kind of risk is common to two or more business units, this

¹⁸ Consider for instance that if one adopted Matten’s suggestion, if the risk free rate is equal to 4% then CaR would be equal to EaR multiplied by 25.

method may fail to capture the compensations between exposure in different business units (imagine the case in which business unit A is exposed to the risk of increasing in interest rates while B has an opposite risk profile). Opposite exposures may in fact be netted correctly only if there is first a groupwide risk mapping and then an aggregation among risk types²⁰.

In theory, therefore, an aggregation across risk types should be preferred at least when the main purpose is to derive the best possible measure of the total capital at risk at groupwide level. Yet, the risk manager often has to handle also risk aggregation across business units, since he is frequently asked to evaluate business units not only according to their undiversified, stand-alone CaR, but also to some measure of diversified capital at risk. Therefore, she or he cannot avoid the problem of estimating correlation coefficients across business units (either directly or indirectly, through the correlation among risk factors and risk factor exposures across business units).

4.2. The aggregation algorithm

Apart from the choice between risk types and business units' aggregation, the risk manager has to choose an aggregation technique and to identify its parameters. As Rosenberg and Schuermann (2004) clearly describe, the portfolio CaR of the whole bank in percentage terms can be written as

$$CaR_{p,\%}(\mathbf{a}) = \mathbf{m}_p + \mathbf{s}_p F_p^{-1}(\mathbf{a}) \quad (11)$$

¹⁹ See Joint Forum Working Group (2003).

²⁰Not surprisingly, yet, aggregating group-wide risk factor exposures becomes more difficult in the case of a financial conglomerate, so that according to the Joint Forum survey they seem to aggregate exposures within all the banking group but excluding the insurance divisions. If we consider for instance a life insurance company and the trading division of a banking group, they are both exposed to market risks, but in a different manner. First, the life insurance company may be sensitive to market risk especially over long time horizons rather than over the short term; secondly, in a life insurance company the market risk may be so intertwined with other typical risks (e.g. deriving from actuarial assumptions made in defining the investment strategy, or from customers' behaviour regarding the surrender or extension options often embedded in life insurance contracts) that it may be necessary to estimate its impact through a joint simulation of all relevant variables.

where $F_p^{-1}(\mathbf{a})$ is the α -quantile of the standardized return distribution. The variance of portfolio return can be written as

$$\mathbf{s}_p^2 = \sum_{i=1}^n w_i^2 \mathbf{s}_i^2 + \sum_{i=1}^n \sum_{j \neq i} w_i w_j \mathbf{s}_i \mathbf{s}_j \mathbf{r}_{ij} \quad (12)$$

where w_i, w_j represent the weights of the portions of the portfolios that are being aggregated, $\mathbf{s}_i, \mathbf{s}_j$ are the standard deviation of their returns and ρ is the correlation coefficient between returns on assets i and j . Therefore, by merging (11) and (12) we get

$$\begin{aligned} CaR_{p,\%}(\mathbf{a}) &= \mathbf{m}_p + F_p^{-1}(\mathbf{a}) \sqrt{\sum_{i=1}^n w_i^2 \mathbf{s}_i^2 + \sum_{i=1}^n \sum_{j \neq i} w_i w_j \mathbf{s}_i \mathbf{s}_j \mathbf{r}_{ij}} = \\ &= \mathbf{m}_p + \sqrt{\sum_{i=1}^n w_i^2 \mathbf{s}_i^2 [F_p^{-1}(\mathbf{a})]^2 + \sum_{i=1}^n \sum_{j \neq i} w_i w_j \mathbf{s}_i \mathbf{s}_j \mathbf{r}_{ij} [F_p^{-1}(\mathbf{a})]^2} \end{aligned} \quad (13)$$

If one assumes that the quantiles of standardized returns of each portion of the portfolio $F_x^{-1}(\mathbf{a}), F_y^{-1}(\mathbf{a}), F_z^{-1}(\mathbf{a})$ are identical to the quantiles of the portfolio returns, then one could substitute $F_p^{-1}(\mathbf{a})$ with the appropriate quantile and express (13) in terms of individual CaR values in percentage terms ($CaR_{i,\%}, CaR_{j,\%}$):

$$\begin{aligned} CaR_{p,\%}(\mathbf{a}) &= \mathbf{m}_p + \sqrt{\sum_{i=1}^n w_i^2 \mathbf{s}_i^2 [F_p^{-1}(\mathbf{a})]^2 + \sum_{i=1}^n \sum_{j \neq i} w_i w_j \mathbf{s}_i \mathbf{s}_j \mathbf{r}_{ij} [F_p^{-1}(\mathbf{a})]^2} = \\ &= \mathbf{m}_p + \sqrt{\sum_{i=1}^n w_i^2 \mathbf{s}_i^2 [F_i^{-1}(\mathbf{a})]^2 + \sum_{i=1}^n \sum_{j \neq i} w_i w_j \mathbf{s}_i \mathbf{s}_j F_i^{-1}(\mathbf{a}) F_j^{-1}(\mathbf{a}) \mathbf{r}_{ij}} = \\ &= \mathbf{m}_p + \sqrt{\sum_{i=1}^n w_i^2 [CaR_{i,\%} - \mathbf{m}_i]^2 + \sum_{i=1}^n \sum_{j \neq i} w_i w_j [CaR_{i,\%} - \mathbf{m}_i] [CaR_{j,\%} - \mathbf{m}_j] \mathbf{r}_{ij}} \end{aligned} \quad (14)$$

A further simplification of the formula could be represented by neglecting the expected return terms (both for prudential purposes and considering their difficult estimation) and by expressing CaR in absolute rather than percentage terms, multiplying both terms for the value of the whole portfolio V . This simply leads to the well-known formula

$$CaR_p = \sqrt{\sum_{i=1}^n CaR_i^2 + \sum_{i=1}^n \sum_{j \neq i} CaR_i CaR_j \mathbf{r}_{ij}} \quad (15)$$

Equation (15) depicts the simplest possible technique for risk aggregation, and corresponds to either Normal VaR or Hybrid VaR, according to the definition of Rosenberg and Schuermann (2004), depending on how individual CaR is calculated. The formula relies on a critical assumption concerning the relation between individual distribution quantiles and portfolio quantiles, that is satisfied in the case of elliptic distributions (a particular case of which is a joint normal distribution). If this assumption is not satisfied, then more sophisticated techniques, such as copulas, would be necessary.

Copulas, in fact, can be much more flexible. Unfortunately, since there are many different possible copula functions, the problem of defining which is better remains. Unfortunately, the risk manager lacks the amount of data that would be needed to support the choice with clear empirical evidence. The choice of the copula function that is best suited to represent the joint distribution of two or more stock indexes' returns is a problem that can be solved by analyzing many years of daily data. When, on the contrary, the problem is the aggregation among business units or risk types, it is virtually impossible to have long enough time series of returns/earnings so as to conduct a similar analysis. Consequently, the choice of a particular copula function would at the end be partially judgmental and arbitrary. This could be a major problem if the aggregated capital at risk measure is expected to enter into the bank's decisional processes. For instance, if business units were evaluated on a risk-adjusted performance measure based on diversified capital at risk, then the aggregation technique may influence the allocation of diversification benefits among different business units, and it could be very difficult for the risk manager to defend a subjective choice against the critiques of

those division manager who feel to be damaged by the specific copula function that has been chosen (at least until a clear “best practice” solution may emerge)²¹.

A second possible alternative solution is the multifactor approach proposed by Alexander and Pezier (2003) who suggested to build a multifactor model for the profit and loss distributions P_1, P_2, \dots, P_n of each business unit, so that

$$P_i = \mathbf{a}_i + \mathbf{b}_{i,1}x_1 + \mathbf{b}_{i,2}x_2 + \dots + \mathbf{b}_{i,n}x_n + \mathbf{e}_i \quad (16)$$

where \mathbf{a}_i is the expected P&L, $\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_n$ are the sensitivities of the P&L of business unit i to changes in the risk factor over the desired time horizon represented by x_1, x_2, \dots, x_n and \mathbf{e}_i is the residual that is not explained by the risk factors. If a linear regression of this kind can be used, then the variance of aggregate P&L can easily be computed. Yet, identifying the scaling factor that may transform the standard deviation into an economic capital estimate requires simulation. Alexander and Pezier (2003) also discuss the possibility of considering a normal mixture distribution for risk factors and of using tail correlations rather than usual correlations to model the dependence between risk factors. A more complex situation would be the case of non-linear relations between the P&L and the factors.

From a practical point of view, the solution they proposed could be particularly appealing if a risk manager could derive a stable scaling factor to translate the standard deviation of the aggregate P&L of the bank into an economic capital measure. In this case, simulation would be used just once (or very infrequently) to derive the scaling factor, and then the standard deviation of aggregate P&L could be derived in closed form from the multifactor model by knowing the \mathbf{b} vector, the covariance matrix of risk factors and the covariance matrix of

²¹ It must be noted that Rosenberg and Schuermann (2004) test empirically which is the impact of different specifications of the copula function by comparing a Gaussian copula with Student-t copulas with 5 and 10 degrees of freedom. They find out that changes in aggregate CaR may be non-negligible in absolute terms – for instance, the difference in CaR between a normal and a Student-t(5) copula is around 11% – even if in relative terms other elements such as the business mix between market, credit and operational risk and the correlation

residual terms. It might anyway be questioned whether (and in case, how long) the mix of businesses and especially P&L's exposures for each business may be considered stable enough to maintain the same scaling factor.

In practice, however, we may suggest a new and fourth solution that many commercial banks may follow, and that is a variant of Alexander and Pezier's proposal. We would label this solution as "mixed multifactor approach". In fact, for most banks credit risk accounts for a huge part of the bank's overall risk. As a consequence, a correct estimate of aggregate capital at risk will absolutely require a correct estimation of stand-alone credit risk economic capital, and the precision in measuring the main risk one should not be sacrificed in order to adopt a simplified model that makes it easier to model correlations²². In this case one could use first the already existing credit portfolio risk model in order to estimate credit risk economic capital: in default-mode models used by commercial banks, this estimate is often derived through a simulation that models the relationship between credit risk losses and some factors representing the state of the economy. It would therefore be possible to extract a value of portfolio losses conditional on a vector of values of the n common factors used for the portfolio model. A multifactor technique *à la* Alexander and Pezier could then be used for the other risks in order to estimate the linkage between the P&Ls of the remaining components of the bank portfolio and a set of m factors. This set would comprise the n factors used for the credit risk model and a further set of $m-n$ factors that may be relevant only for the other kinds of risk. Then for each of the k scenarios simulated by the credit portfolio risk model – that has a specific vector of the n credit factors and a specific credit P&L – it would be possible, given

between operational risk on one hand and market and credit risk on the other hand have a greater impact on the aggregate risk estimate.

²² It must also be considered that - as Kuritzkes, Schuermann and Wiener (2002) clearly point out - when the overall risk is dominated by a single type of risk, diversification benefits become smaller. Therefore, the overall CaR may be more sensitive to errors in modelling the single major risk than to errors in estimating correlation coefficients.

the dependence structure between risk factors, to extract a correlated vector of the remaining $n-m$ factors (and of course of the residual terms \mathbf{e}_i for each business unit) and to derive a simulated P&L for all the remaining business units/risk portfolios into which the bank's overall portfolio has been divided²³. Using this two-step simulation approach may be computationally intensive, but may have advantages when credit risk is particularly significant. More specifically, this approach:

1. uses the more sophisticated credit portfolio risk model (with its typical non-linear relationships between factors and portfolio losses) instead of an approximation in order to estimate one of the biggest parts of the bank's capital at risk;
2. does not duplicate the simulation effort on the credit portfolio, since running the simulation is useful both for credit risk management purposes and for estimating aggregate risk capital²⁴, and it does not require to spend time estimating betas for credit risk;
3. derives capital at risk simply as the desired percentile of the distribution of the aggregate P&L of the bank, obtained by simply adding up business units' P&Ls in the same scenario;
4. since it reconstructs the full distribution of P&L values, it may also enable to estimate (provided the number of simulations is high enough) other risk measures which can

²³ Of course, for all the other parts of the bank's portfolio the choice whether to use a simplified multifactor model or a more sophisticated model (consider for instance the case of market risk) will depend on an analysis of the relative relevance of the risk at an aggregate level and the cost of developing or using a more sophisticated model. The key requisite is to use a simulation approach across all risks and to condition the $m-n$ values of the remaining common risk factors on the n values of the factors extracted for the credit risk portfolio model.

A very similar approach has been recently proposed also by Dimakos and Aas (2003), and implemented at Den Norske Bank. Even in this case market and operational risk are estimated conditional on credit risk, but the authors suggest – as a declaredly simplifying assumption – to model the dependence on credit risk only through the relationship with the frequency of defaults, instead than through macro factors (including interest rates) to which simulated defaults are typically linked in a credit risk VaR simulation.

²⁴ Instead, by using a factor model, it would probably be necessary to run both the “extended” credit risk model, for credit risk management purposes, and the “simplified” one for risk aggregation purposes. Therefore however simple the latter model may be it would cause a net increase of computational work.

complement (or maybe substitute) the capital at risk measure in more sophisticated approaches.

All the aggregation techniques that we have considered have of course to face the relevant problem of parameters estimation. Before evaluating their overall pros and cons it is therefore useful to deal with this issue in more detail.

4.3. The issue of parameters' estimation for different risk aggregation techniques

Parameters' estimation is a critical issue. In fact, even if the simple equation (15) formula were adopted, the ρ coefficients might be estimated in completely different ways. Basically, four different solutions may be adopted:

- 1) defining correlation coefficients in a judgmental way;
- 2) deriving historical correlation coefficients from publicly available stock return series of listed financial institutions;
- 3) deriving historical correlation coefficients from internal earnings series of different business units;
- 4) deriving correlation coefficients between risks through a simulation of existing internal risk measurement models.

According to the Joint Forum Working Group (2003), the first solution is already used by some institutions, which determine coefficients judgmentally, either completely (through a subjective evaluation made by top managers, or by resorting to coefficients used in one of the few existing papers on the subject²⁵), or in part, by blending empirical coefficients with judgmental ones²⁶. A judgmental evaluation may be questionable, and may be influenced by the fact that many potential "judges" inside the bank may be business units' managers whose

²⁵See in particular Kuritzkes, Schuermann and Wiener (2001), Ward and Lee (2002).

results might be affected by the final estimates on correlation; this might alter, of course, the outcomes of an internal panel on “perceived” correlation.

Since it would be therefore desirable to extract correlation coefficients by historical data, a possible solution would be to observe stock returns correlation of stand-alone, monobusiness (or mono-risk) financial institutions. If they existed, the task would be easy, but it is very difficult in practice to find - for all the possible divisions of a real bank - competitors that can be considered specialized enough. While in theory it could also be possible to try to derive common factors (e.g. through principal components analysis) from return series of non-specialized financial institutions, associating common factors to individual risk types or internal business units/divisions would probably be too subjective for a topic that may be so relevant such as risk capital aggregation and allocation.

The typical solution is therefore the third one, i.e. estimating correlation between earnings’ time series of different business units, as suggested in Matten (1996). Nevertheless, there are at least four problems in using this method:

- historical correlation coefficients may simply be impossible to derive when business units have recently been restructured (e.g. due to a merger) so that the historical time series of returns that is available is either nonexistent or far too short;
- available time series are typically series of earnings, not returns, and translating them into return series may be in some cases partially arbitrary since it requires a measure of “exposure” to the risk²⁷;

²⁶See Joint Forum (2003), pp. 24-25, that reports that some banks also use an average between empirical correlation coefficients and one, in order to make the estimate more conservative.

²⁷ The problem has already been pointed out in section 2. Of course if the exposure of the different business units is considered to be fixed, then correlation among returns and earnings is the same since both earnings’ series are just scaled by a constant. If instead exposure is considered to be variable (imagine the case of a business unit that experienced through the sample period a significant dimensional growth) then the choice of the exposure measure becomes much more relevant.

- they are typically based on business units' P&L's, so that – if individual CaR estimates were based on risk type – business units' correlation coefficients would only provide a proxy for the required correlation, since it would be necessary to associate a risk type to the business unit where the single risk is predominant;
- if (in order to increase available data despite short time series) they are calculated on infra-annual and especially on monthly data, there are good reasons to expect that historical correlation coefficients can provide a downward biased estimate of “true” correlation due to the existence of serial cross-autocorrelation in returns.

While the first and the second problem are quite simple and usually well recognized by risk managers, the third one is often not perceived and deserves a clearer explanation. Simply speaking, when a monthly series of P&Ls of the different divisions is used in order to estimate earnings correlation, if the earnings of the business unit X in month t are correlated not only with the earnings of business unit Y in month t , but also to its earnings on month $t+1$, $t+2$, and so on, then the correlation estimate may prove to be wrong. We will first show (see Table 1) a purely theoretical example of the effects of serial cross-autocorrelation between earnings series, and then explain why this effect is likely to occur in practice.

Let us consider business units X and Y whose monthly earnings are reported in Table 1. According to those data, the correlation ρ between monthly earnings is close to zero. Yet, it can be easily seen that from the third month onwards the earnings of Y are simply equal to the average of the earnings of X in the two months before. In practice (see the arrows on month 5) this is equal to assuming that an event that has direct and immediate influence on X's earnings on month t impacts Y's earnings partly in month $t+1$ and partly in month $t+2$ (and there is therefore serial cross-autocorrelation). It is no surprise, therefore, that quarterly correlation results to be markedly higher than monthly one (in the example, it is 0.675 instead

of 0.001). Please note that while the example is related to earnings series for sake of simplicity, the same result would occur with business units returns if the size of exposure in X and Y were constant through the sample period (in fact, returns for both units would be equal to earnings divided by a constant, and the correlation would be unaffected by such scaling).

Table 1. An example of the problem of earnings cross-autocorrelation

	Monthly earnings		Quarterly earnings	
	X	Y	X	Y
1	10	-200	120	-205
2	-30	5		
3	140	-10		
4	-70	55	100	25
5	-60	35		
6	230	-65		
7	200	85	430	470
8	140	215		
9	90	170		
10	80	115	490	370
11	260	85		
12	150	170		
13	60	205	100	360
14	40	105		
15	0	50		
16	190	20	390	340
17	260	95		
18	-60	225		
19	-60	100	-50	45
20	70	-60		
21	-60	5		
22	130	5	300	180
23	150	35		
24	20	140		
Correlation	$r_{monthly} = 0.001$		$r_{quarterly} = 0.675$	

Therefore, earnings’ serial cross-autocorrelation may make completely untrustworthy correlation coefficients that are calculated on short-period earnings data, which may fail to capture the substantial interdependence between the two business units/areas²⁸. Why then

²⁸ The effect, by the way, is similar to the one that characterizes the estimates of correlation coefficients between, say, stock indexes or between individual stocks, when there is serial cross-autocorrelation between daily (or weekly) returns, so that correlation coefficients based on monthly data may be remarkably different from the ones extracted by daily data.

should we suspect the existence of earnings serial cross-autocorrelation? There are at least two reasons to do that.

First, this phenomenon may derive from the different accounting practices used for the profit centers inside the bank. A rise in interest rates may represent bad news both for the Treasury bond trader and for the head of the asset and liability management strategies of the banking book (e.g., because there is a negative maturity gap so that there are more interest-sensitive liabilities than assets). Yet, while the effect of the adverse rate move would impact immediately the bond portfolio which is marked to market, it would influence the profit of the banking book only gradually, through a reduction of the net interest margin in the following months.

Secondly, earnings serial cross-autocorrelation may also derive from structurally different reactions of different businesses to the same event. Consider the impact of a sharp stock market drop. In the equity trading division this may produce immediate losses, which are promptly recognized in the profit and loss of the profit center. The same event may have instead diluted but prolonged effects on areas such as the investment banking division (since firms' potential equity offerings may be rescheduled or suppressed) and the asset management division (due to the reduction of asset under management and, consequently, management fees, and to the indirect effect of the switches that may take place from equity to bond mutual funds, reducing the average level of fees for the asset manager). In both cases, the effects of the stock market fall in month t may be material even several months after.

Hence, since there are reasons to suppose that earnings serial cross-autocorrelation may be frequent, the risk manager has to choose in practice whether to estimate correlation coefficients between different business units' earnings using an apparently larger, but less

trustworthy, sample of monthly data or instead aggregate earnings over longer time intervals accepting to calculate correlation coefficients with very few couples of earnings data.

A fourth possible solution could then be to derive correlation coefficients by simulating scenarios using existing internal risk models (e.g. the credit portfolio risk model, the market risk model...). Since they define potential losses as a function of a set of different factors (e.g. market factors for the market risk model, macroeconomic or other common factors for the credit risk portfolio model), it would be possible to produce a joint simulation of the different risk models (of course under a consistent set of values of the key factors) and then derive correlation coefficients from the simulated scenarios. Yet, adopting this solution to derive linear correlation coefficients for equation (15) has two clear shortcomings. First of all, risk models are available only for some of the risk types or business units. Secondly, if they existed and it were possible to run a joint simulation then a mixed multifactor approach would be much better to aggregate CaR than using the simplified equation (15).

The problems for parameter calibration that make it difficult to estimate linear correlation coefficients may be applied also to the estimation of the parameters of copula function, where a longer series would be even more desirable. In practice, historical data would fail to provide an adequate dataset and model-generated scenarios would appear at present the only possible solution; but, again, they are not available for all risk types; moreover, once scenarios are generated, losses could be aggregated directly. Therefore, despite the critique of usual linear correlation coefficients that lies behind the growing interest in copula functions is precious and well-founded in the field of risk aggregation, it remains unclear whether copulas

themselves may be at present a sufficiently transparent solution to the problem of risk aggregation²⁹.

In the multifactor approach *à la* Alexander and Pezier (2003) and in the mixed multifactor approach the dependence structure among different business units is driven by (1) the existence of common factors that impact individual P&Ls and (2) the dependence structure among common factors. It is therefore critical to correctly identify the common factors and their relationship with the P&L of each business unit. In the “pure” multifactor approach this aim could be attained either through historical earnings data or through the risk model for those kind of business units where a clear risk model is already in place, such as those mainly exposed to credit or market risk, while historical earnings’ analysis is likely to be the only possible solution for business units that are mainly exposed to business risk. This is true also for the mixed multifactor approach, with the only remarkable difference that there is no need to develop estimates at least for credit risk and for those business units that are mainly exposed to credit risk³⁰. The problems deriving from the absence of long enough earnings data series and from the likely presence of cross-autocorrelation in earnings data when they are measured on short intervals still apply, and may impact the estimate of P&L’s sensitivities to common risk factors; yet, if a mixed multifactor approach is adopted, these problems are confined only to a share (even if certainly not negligible) of the overall business units/risk types, and therefore their impact may be reduced. A critical issue for both approaches is in

²⁹ To some extent, yet, one could consider the mixed multifactor approach that aggregates values inside each scenario as a particular way to use an extremely precise empirical copula estimated through model-generated data. In any case, no formal known copula function parameters should be estimated in this case. Copulas may instead be useful to model the joint distribution of the common risk factors and to correctly reproduce their dependence structure when simulating scenarios.

³⁰ The case of market risk, where there is typically a clear risk model with a huge number of different risk factors (e.g. different exchange rates, different zero rates of different curves, and so on), may be treated in a different manner depending on the characteristics of the individual bank. There might be the need in fact to reduce the number of relevant factors to a subset of initial risk factors in order to speed up the simulation. The reduction of the number of factors that can be achieved obviously depends on one hand on how concentrated the exposure is

any case the choice of the number of factors and the way in which residual risk is modelled. In fact, since aggregated CaR is determined by simulation, it could be desirable to be parsimonious in the choice of common risk factors. The lower the number of common risk factors the faster and less complex is the simulation, but at the same time the more critical becomes the way in which residual risk is defined. In theory, if there were a large number of entities for which a P&L is calculated having a reasonably similar size and if residual components were uncorrelated then one could argue that the residual components could be irrelevant due to diversification. In practice, yet, some divisions are likely to account for a substantial part of the overall risk, and if the number of common factors is limited then the residual components may not be completely independent. In this case, modelling the variance and dependence structure of these components (at least for larger business units) is very important.

Finally, the dependence structure of common factors is also important. Assuming joint normality and simulating correlated risk factors through a simple Cholesky decomposition would be the easier path to follow, but this assumption may not be the best one for all risk factors. In this case, more sophisticated techniques (and even copulas, applied to risk factors instead than to earnings distribution directly) may be desirable, provided that data series related to selected risk factors are sufficient to support the estimation of their parameters.

4.4. Deriving measures of diversified risk capital for individual business units

A problem which is seldom considered is the ability of the risk aggregation method to produce consistent estimates of “diversified” capital at risk for individual business units. The risk manager may in fact need to measure the risk-adjusted return of a certain part of the bank

on some key risks, and on the other hand on the overall share of market risks on the total CaR of the bank, that may influence the impact that simplifications may have on the aggregated CaR estimate.

keeping into consideration not only stand-alone CaR, but also a measure of its contribution to overall capital at risk. This typically implies evaluating the business area or the business unit with a lower CaR number that takes into account diversification benefits³¹ (since CaR is not subadditive, it is not 100% certain that the measure is going to be lower than stand-alone CaR). It is therefore relevant to check whether the methodology for risk aggregation is able to produce in a consistent manner both a measure of overall aggregated CaR and measures of individual, diversified CaR. Otherwise, apart from the duplication of workload for the risk manager, the overall risk capital measurement technique might become with how individual business units are evaluated in practice. Capital management decisions at bank level and capital allocation decisions within business units would rely on different methodologies.

Let us consider first the case in which the simple equation (15) is used. In this case it is quite simple to derive any possible measure of diversified CaR, such as marginal CaR (i.e., the CaR that can be saved if a given business unit or business area were eliminated from the portfolio) or even CaR diversified through the portfolio method ($CaR_{div,i}$), which is equal to

$$CaR_{div,i} = CaR_{und,i} \cdot r_{i,b} \quad (17)$$

where $CaR_{und,i}$ is the undiversified, stand-alone CaR of the business unit/area³² i , and $r_{i,b}$ is the correlation coefficient between the returns produced by the business unit i and the bank's overall return. The advantage of this method is that it is additive, i.e. the sum of all diversified CaR values equals overall CaR, which is desirable since it simplifies the definition of return targets and ex post performance evaluation. In this case, once the correlation coefficients in

³¹ There are different ways to produce a diversified measure of capital at risk: see for instance James 1996, Matten 2000, Merton and Perold 1996, Saita 1999.

³² The term “undiversified” means here that diversification benefits (if they exist) deriving from the interaction between the business area/unit and other business areas/units are not considered. Of course, diversification effects *inside* the business units (e.g. among different kind of risks) are already taken into consideration.

equation (17) have been derived to determine aggregated CaR, values for $r_{i,b}$ for each business unit can be easily derived too, and the method is perfectly consistent.

As far as copulas are concerned, if they are used to aggregate the effects of different kind of risks, it could be questioned whether the same copula function should be used every time that, for instance, market and credit risk are aggregated, both at bank level and inside a given business unit, or instead whether the copula that best fits the joint distribution should be used from case to case. In the second case some problems of consistency may emerge.

In the case of the two other methods, deriving consistent individual CaR measures (both diversified and undiversified) is easy if the simulation is properly structured, and it is possible to save in each scenario not only the value of the big portfolios that are aggregated (i.e., the credit risk portfolio, the aggregate market risk exposure, and so on) but also the value of its major subcomponents (i.e., the value of the part of the credit portfolio attributed to unit A, or B, or C). In this case in each scenario it is clearly derived both an overall bank portfolio value, whose desired percentile would define the bank's overall capital at risk, and an individual portfolio value, from whose distribution an undiversified measure can be easily derived. A possible solution to derive an additive diversified CaR measure is then to resort to the concept of co-measures developed by Kreps and described by Venter (2003), where the co-capital at risk of business unit i is defined as the average value of portfolio i (X_i) when the overall bank portfolio X is at the desired quantile q :

$$co - CaR_i = E(X_i | F(X) = q) \quad (18)$$

In practice, co-CaR of business unit i would be equal to the expected value of the losses in the business unit i when overall bank losses are equal to the bank's capital at risk at the confidence level defined by q . This result can be constructed as a by-product of the overall simulation, and the sum of all the co-capital at risk values would equal the value at risk at the

desired percentile level. Therefore, it would be possible to derive both a diversified (and additive) and an undiversified measure of capital at risk in a manner which is totally consistent with the overall methodology adopted for risk aggregation.

4.5. A synthetic analysis of alternative risk aggregation techniques

So far we have noted that evaluating the different methodologies for risk integration requires both to consider the theoretical properties of the methodology and the practical issues that arise when trying to estimate its parameters. There are, therefore, a number of different criteria that should be considered when evaluating a risk integration methodology. Some of them are objective and structural, others are still objective but may become less important through time (e.g. the ability to reduce sampling errors or model risk due to the absence of adequate time series of earnings). Still, others are subjective since they depend on the purposes for which the risk manager will use the aggregation technique.

For instance, attributes such as transparency, fairness, or the ability to produce diversified CaR measures, may have a relevance that varies substantially depending on whether the risk aggregation exercise is intended to support only top management's decisions on capital structure or also to guide the internal capital allocation process. This also means that choices of different techniques made by different banks might find a rationale in the different breadth of purposes to be pursued.

Table 2 – A comparison of the different risk aggregation techniques (normal font is used for general advantages or disadvantages, while italic highlights calibration issues)

Methodology	Advantages	Disadvantages
Classic square root formula (Normal CaR or H-CaR)	<ul style="list-style-type: none"> • Simple, intuitive and explainable to non-experts • Aggregated capital at risk can be derived analytically by individual CaR figures and correlation coefficients • Diversified CaR figures for individual business units are easy to calculate • <i>Parameter estimation is technically simple (even if subject to major data problems)</i> 	<ul style="list-style-type: none"> • Theoretically incorrect when the standardized quantiles of the portfolio distribution are not identical to portfolio standardized quantiles • <i>Estimating correlation through historical earnings' series may not capture "current" correlation for some kind of risks (i.e., market vs. credit risk)</i> • <i>Lack of long series of data may lead to use short term (e.g., monthly) earnings that may lead to underestimate correlations if there is serial cross-autocorrelation</i>
Aggregation through copulas	<ul style="list-style-type: none"> • It is not necessary to assume a particular joint distribution (e.g elliptic) of returns/earnings for all risks/business units • The choice of the right kind of copula function can enable to model dependence (and also tail dependence) correctly while maintaining the original distribution of returns/earnings for the single risk/business unit • 	<ul style="list-style-type: none"> • The method is more complex and harder to explain to non-experts. • If used also for allocation to individual business units, diversified CaR figures might be very sensitive to the choice of the copula function and its parameters, increasing political resistance • It may be more difficult to develop consistent diversified CaR measures for individual units • <i>The choice of the optimal copula function and its parameters requires a significantly long series of earnings data that is surely missing in most banks. Shorter series may lead to a substantial model risk or at least to a higher subjectivity of the choice of the copula function and its parameters.</i>
Multifactor approach à la Alexander and Pezier (2003)	<ul style="list-style-type: none"> • Intuitive approach that forces to understand better the common factors behind different business units. • If CaR is assumed to be a fixed multiple of portfolio standard deviation, then aggregated CaR can be derived analytically. • Interesting solution to model correlation among fee-based, "business risk prone" business units and between those business units and the others. • The method can be consistent with different levels of complexity in modelling risk factor returns and their relationship with P&Ls 	<ul style="list-style-type: none"> • If normality of portfolio returns is not assumed, the right multiple has to be estimated (and periodically re-estimated) through simulation. • The method is likely to simplify the relationships between factors and business units' P&L also for those business units who already possess a more sophisticated model. This can impact overall results especially if some of those business units account for a significant part of the overall CaR. • <i>It is necessary to carefully select the number and type of risk factors, and correctly estimating also the specific risk component especially for larger business units.</i> • <i>Estimating factor sensitivities is exposed to problems due to lack of long time series (as for other alternative methods).</i> • <i>The dependence structure between different risk factors may be critical.</i>
Mixed multifactor approach	<ul style="list-style-type: none"> • The precision of the credit risk portfolio model, which often accounts for a relevant share of overall CaR, is completely preserved. • Aggregation occurs through simulation in each simulated scenario, and is therefore extremely straightforward and easy to explain to non-experts. • It is possible to derive diversified and undiversified measures of CaR as a by-product of the simulation • <i>Factor sensitivities must be estimated only for those business units which are not exposed to market and credit risk, that are already captured by existing models</i> 	<ul style="list-style-type: none"> • Computationally intensive • <i>It is necessary to carefully select the number and type of risk factors, and correctly estimating also the specific risk component especially if there are large business units whose risks are not captured by the credit or market risk models.</i> • <i>Estimating factor sensitivities is exposed to problems due to lack of long time series (even if this problem applies only to a subset of business units which may not be the larger ones).</i> • <i>The dependence structure between different risk factors may be critical.</i>

The advantages and disadvantages of the four methods that have been discussed are reported in Table 2. Calibration problems are stressed since in practice the difficulty to feed the risk aggregation technique with reasonable numbers may be a major obstacle in its adoption. This is true in particular when the same numbers will be used also to evaluate individual units. On one hand there are more stakeholders who may be willing to question the procedure used to derive those numbers. On the other hand, if individual risk measures are used to drive or to support decisions on the business mix and the internal allocation of resources, then estimation errors may be a more serious problem. A significant amount of caution is therefore required when bringing the aggregation technique and its formula from the R&D laboratory to the plants, and – at least in early stages of development – a risk manager might decide not to use the technique to also evaluate diversification effects inside the bank and their attribution to business units.

Moreover, when a sophisticated technique is fed with a very short series of historical data (e.g. monthly earnings of certain business units), model parameters may change substantially from one year to another, simply because of the increase in the size of the historical sample. It would then be problematic to provide top managers with a report that could contain major changes in overall and individual CaR values even in the absence of similar changes in business units' activity. Consequently, it may be advisable to test how the results of different methods change when new data are added, especially if the sample is not totally trustworthy. In general, as Pezier (2003) points out, it is important to make clear to all of the potential users that single CaR numbers are estimates and often very uncertain ones, and cannot drive strategic choices in any mechanical way.

4.6. Implications for risk-adjusted performance measurement

The risk aggregation problems that have been discussed have interesting implications also for risk-adjusted performance evaluation of individual business units. Let us consider initially the problem of risk measure dishomogeneity discussed in Section 3. First, if risk measures are not perfectly homogeneous then comparisons of RAROC or EVATM values of individual divisions should be considered with great caution, and methodological differences and their likely impact on numbers and relative rankings must be clarified. Clearly, top managers may not share the technical skills needed to capture all the subtleties of the differences between individual risk measures' methodologies. Clarifying the key assumptions and the key differences between the measures that are being compared is an aim that could hardly be achieved simply by producing detailed technical documents. These documents, albeit precious for other users, are typically inadequate for a member of the board or a senior top manager and may instead reinforce a sense of statistical soundness and intrinsic precision that is often misplaced.

Secondly, the relative ranking of business units may be also sensitive to the apparently purely technical choice of the confidence interval and of the rescaling technique that is adopted. Since CaR values do not grow proportionally for different types of risks/business units as the confidence level increases, then RAPM parameters would change too. This problem could be reduced if the adopted risk metric considered the entire adverse tail of return distribution. An example of such a risk measure is expected shortfall, defined as the average loss in the worst $\alpha\%$ (e.g., 5%, 10%) of possible cases³³. While such a measure remains sensitive to α , it is not going to be so sensitive as VaR, since increasing α implies considering less and less extreme

³³ We adopt here the term of expected shortfall even if different terms are sometimes used in his place, such as tail VaR, tail conditional expectation, and so on. Acerbi and Tasche (2001) discuss how these measures can be identical under continuous return distributions but differ under discrete ones.

cases. Yet measuring expected shortfall in practice throughout the bank, and especially for those businesses where data are often so scarce that even a simple variance estimate is highly uncertain, is far from easy at present.

A third and largely neglected point is the impact that differences in the notion of capital may have both on ex post performance evaluation and on ex ante target setting. This is critical especially for banks when the exposure to business risk (e.g. through asset management divisions, or certain corporate finance services units) is relevant. Section 2 has shown that a market cap CaR can substantially differ from a measure of book-valued capital at risk, and obviously the contribution of business-risk-prone fee-based divisions would radically change depending on the perspective that is adopted. What is perhaps less obvious is that not considering market cap CaR for those units could also alter the definition of target returns.

Imagine the case of a bank whose market capitalization is equal to 150 and whose book value of capital is equal to 100, and has therefore a price to book value equal to 1.5. A bank that using the CAPM estimates its cost of equity capital k_e at 10% would not satisfy its shareholders if ROE were 10%, since its price to book ratio is higher than one. In fact, the cost of equity capital is how the bank should remunerate an amount of capital equal to its market capitalization. If the book value is lower than the market cap, then ROE should be proportionally higher. This implies that if the bank tries to set RAROC targets according to the desired return on a CaR measure defined in terms of book value, it might be forced to raise RAROC targets over 10% to compensate for the fact that market capitalization is higher than book capital. If instead internal capital allocation and RAROC setting were based on a measure of “market capitalization at risk”, as discussed in Section 3, then allocated capital would result to be higher and a lower RAROC target could be set. While at bank level a final equilibrium may be reached, at individual business level the choice of the method may impact

the level of the target. Obviously, those business units for which “book CaR” and “market cap CaR” are closer would prefer the first solution, that leads to an increase in RAROC targets that is equal for all units; the others, instead, would favour the opposite solution, that may increase RAROC targets for those units who may be contributing more to explain the bank’s price to book value while leaving the targets for the others unchanged.³⁴.

5. Final remarks

Risk capital aggregation is of paramount importance both for banks and for regulatory authorities. For a bank, it can be very important both to support decisions concerning the overall endowment of capital that may be needed, keeping under consideration the minimum regulatory requirements, and – as it is seldom observed – to derive a technique to consistently measure the contribution of each business unit to the overall risk and support the capital allocation process. From the regulator’s perspective, analyzing the methodology the bank is using to evaluate overall CaR and the calibration of its parameters may be a relevant issue in the supervisory review process. Mistakes in internal estimates about the real amount of capital which is needed and about the contribution of different business units may become a threat to the bank’s survival. It could also be questioned to which extent aggregation techniques should be disclosed, since on one hand they can represent a significant advantage of a bank in supporting its business decisions, while on the other hand they are a relevant piece of information for the market. Bank equity analysts may in fact be given numbers concerning

³⁴ For sake of simplicity, we pointed out the problem in a very simplified way without adding a number of other relevant factors, such as taxation, the opportunity to differentiate target returns according to the business unit’s contribution in determining the bank’s beta and therefore the cost of capital, the possibility that some business units may be represented by legal entities that are not entirely owned by the bank holding company, the question whether the correction is made by adding up individual CaR values considering or ignoring diversification benefits, just to quote a few.

individual risk measures and diversification benefits but may have no way to infer through which assumptions and methodology those numbers have been produced.

While risk aggregation is so relevant, the risk manager is asked to produce measures that might support relevant decisions lacking both a fully agreed, best practice methodology and, perhaps most importantly, lacking an adequate set of data. In this paper we suggest alternative solutions to include business risk in the overall risk measure in a more consistent manner. We also suggest a variant to recently proposed aggregation techniques and we point out the hidden risks in measuring correlation through short-term earnings data. However, it is evident that the development of the field could not rely on the development of more sophisticated aggregation methodologies only. Greater efforts are needed in producing higher quality data to support the existing methodologies, and to increase top managers' understanding about the underlying assumptions of the measures that are reported. Only in this way the risk measures produced may avoid to become either irrelevant in the decisional processes, as it would happen if they were perceived to be untrustworthy, or even dangerous, if they were perceived to be perfect even when they are still far from it.

References

- Acerbi C. and D. Tasche (2001), *On the Coherence of Expected Shortfall*, working paper.
- Alexander C., and J. Pezier (2003), *On the Aggregation of Market and Credit Risks*, ISMA Centre Discussion Papers in Finance 2003-13, University of Reading, October.
- Artzner P., Delbaen F., Eber J.M., Heath D. (1999), "Definition of Coherent Measures of Risk", *Mathematical Finance*, n.3, 203-228.
- Basel Committee for Banking Supervision (2003), *The New Basel Capital Accord*, consultative document, april.
- Culp C. (2000), "Raroc Revisited", *Journal of Commercial Lending and Credit Risk Management*, March, pp. 50-57.

Dimakos X.K., K. Aas (2003), *Integrated Risk Modelling*, working paper, Norwegian Computing Center, December.

Embrechts P., A. McNeil, D. Straumann (1999), *Correlation and Dependence in Risk Management: Properties and Pitfalls*, Risk Lab Working Papers, August.

Froot K.A., J.C. Stein (1998), "Risk Management, Capital Budgeting and Capital Structure Policy for Financial Institutions: An Integrated Approach", *Journal of Financial Economics*, vol. 47, n. 1, January, pp. 55-82.

Hall C. (2002), "Economic Capital: Towards an Integrated Risk Framework", *Risk*, October, pp. 33-38.

James C. (1996), *RAROC Based Capital Budgeting and Performance Evaluation: A Case Study of Bank Capital Allocation*, Wharton Financial Institutions Center Working Paper.

Joint Forum Working Group (2003), *Trends in risk integration and aggregation*, Bank for International Settlements, Basle, August.

Kuritzkes A., T. Schuermann and S.M. Weiner (2001), *Study on the Risk Profile and Capital Adequacy of Financial Conglomerates*, Oliver, Wyman & Co. (available at http://www.dnb.nl/publicaties/pdf/study_risk_profile.pdf)

Kuritzkes A., T. Schuermann and S.M. Weiner (2002), *Risk Measurement, Risk Management and Capital Adequacy in Financial Conglomerates*, Wharton Financial Institutions Center Working Papers Series, n. 03-02.

Matten C. (1996), *Managing Bank Capital, Capital Allocation and Performance Measurement*, First edition, John Wiley & Sons, Chichester.

Merton R., Perold A. (1993), "Theory of Risk Capital in Financial Firms", *Journal of Applied Corporate Finance*, Vol. 6, No. 3, pp. 16-32.

Perold A.F. (1999), *Capital Allocation in Financial Firms*, working paper, Harvard University, Graduate School of Business Administration.

Pezier J. (2003), *Application-Based Financial Risk Aggregation Models*, ISMA Centre Discussion Papers in Finance 2003-11, University of Reading, September.

Rosenberg J.V., T. Schuermann (2004), *A General Approach to Integrated Risk Management with Skewed, Fat-Tailed Risks*, Federal Reserve Bank of New York Staff Report, n.185, May.

Saita F. (1999), "Allocation of Risk Capital in Financial Institutions", *Financial Management*, Autumn, pp. 95-111.

Venter G.G. (2003), *Capital Allocation: An Opinionated Survey*, CAS Forum Summer 2003 (available at <http://www.casact.org/pubs/forum/03sforum/03sf279.pdf>)

Wang S. (2002), *A Set of New Methods and Tools for Enterprise Risk Capital Management and Portfolio Optimization*, CAS Forum Summer 2002 (available at <http://www.casact.org/pubs/forum/02sforum/02sf043.pdf>).

Ward, L.S. and D.H. Lee (2002), *Practical Application of the Risk-Adjusted Return on Capital Framework*, CAS Forum Summer 2002 (available at <http://www.casact.org/pubs/forum/02sforum/02sftoc.pdf>).